Reliability Enhancement of Smart Metering System Using Millimeter Wave Technology

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Abstract—Millimeter wave (mmWave) technology has been advocated as a promising infrastructure to provide reliable communications, both in indoor and outdoor environments. In this paper, we extend the application of mmWave to the uplink communication between smart meters (SMs) and a gateway. Such a communication is subject to interference from SMs belonging to adjacent networks and blockage caused by human bodies. Using a three-dimension (3D) stochastic blockage model, we derive the outage probability. When human-body blockage is neglected, the high signal-to-noise-ratio (SNR) analysis shows a diversity gain of \( (m_L M) \), where \( m_L \) is the Nakagami-fading parameter of the line of sight (LOS) of the reference transmitter’s channel and \( M \) is the number of receive antennas at the gateway. Accounting for human-body blockage, the diversity gain reduces to \( (m_N M) \) where \( m_N \) is the Nakagami-fading parameter of the non-line of sight (NLOS) of the reference transmitter’s channel. Our analysis shows that the probability that an SM is in LOS decays exponentially with the link length and the density of blockages. Although at high SNR blockage reduces the diversity gain, our numerical results show that blockage may decrease the outage probability at finite SNR.

Keywords—mmWave propagation channel, indoor radio channel, Nakagami fading, shadowing, Poisson point process.

I. INTRODUCTION

A. Motivation and Background

Smart Grid (SG) is considered as an intelligent network for delivering electrical power systems [1]. It combines the transmission of electricity and information in order to improve the quality of electricity transmission [2]. It includes a variety of operational and energy measures including smart meters (SMs). Generally, smart metering system (SMS) consists of metering and communication infrastructures [3]. The communication infrastructures of the SG may include wireline technologies such as power line communication (PLC) or wireless communications. However, the PLC faces many technical challenges like the unexpected propagation characteristics of transmission and distribution lines, strong electromagnetic interference and higher signal losses [3]. Hence, wireless communication has been incorporated in the SG as it brings several advantages in terms of installation, coverage, and high flexibility.

Standards such as Worldwide Interoperability for Microwave Access (WiMax), Universal Mobile Telecommunications System (UMTS), Long Term Evolution (LTE) and LTE-Advanced (LTE-A) have been already used for SG communications, in outdoor environments. Likewise, standards such as IEEE 802.11, IEEE 802.15 based Wi-Fi, and Wireless Personal Area Network (WPAN) have been used for indoor environments. Also, several industrial standards are based on IEEE 802.15.4 in order to perform monitoring and control applications. One of the most widely adopted standard in this class is ZigBee due to its extended network capabilities [4].

However, despite its numerous advantages, wireless communications in the SG suffer from many challenges such as limited bandwidth and sensitivity to interference [5]. In that sense, the transmission at mmWave frequencies seems to be a promising alternative for SG due to its immunity against interference and its free wideband spectrum [6]. In addition, the management of SG power resources have the following requirements: Real-time processing of data, and lower transmission latencies with reliable communication over the network [7]. Evidently, mmWave technology meets these requirements as one of the available technology options for reliable, secure, and cost effective operation of SG [7]. MmWave is an element of 5G technology, and in [8], the authors showed how 5G mobile cellular network provides an adequate environment for monitoring and control tasks in smart grid. In other words, they showed that smart grids need a wide area monitoring system (WAMS) to detect and counteract power grids disturbances in real time, thus requiring a communication infrastructure able to:

- Integrate measurement devices like smart meters (SMs) with extreme communication reliability and ultra-low (millisecond) latency.
- Provide support for distributed and real-time computation architecture to provide an estimate of the system state variables, i.e., voltage, and current magnitude.

In [9], two scenarios have been compared: an LTE-based centralized network management approach and a 5G-based distributed network management approach in the smart grid. The 5G-based distributed network shows a significant improvement over LTE-based distributed network in the performance of the network management by reducing the latencies which is beneficial for the monitoring and control of the smart grid [9]. To summarize, mmWave meets two essential requirements of the SMS:

- Real time-processing of the data and lower transmission latencies.
- High-reliable communication over the network.

Note that, since the usual data rate requirement in SMS is around 300-500 kbps [10], communications over mmWave frequencies is implemented for SMs only for its desirable...
lower latency and higher reliability, and not for high data rates requirements. Furthermore, SMS applications are short range in nature which is suitable to mmWave technology. There are two commercial standards such as Wireless HD [11] and IEEE 802.11ad [12] which are dedicated to applications using mmWave technology. Since it targets indoor environments, the last standard could be a candidate for SMS.

B. A Literature Review

Recent works have investigated mmWave networks using stochastic geometry tools to analyze the coverage probability [13]. In [14], a closed-form expression was derived for the outage probability conditioned on the network geometry, in indoor wearable settings. Also, [14] has analysed a finite network with finite number of interferers. In [15], a similar approach as in [14] has been adopted to obtain the outage probability conditioned on the network geometry in Ad Hoc networks. An extension of such an approach to frequency-hopping network considered in [16]. Although there has been considerable progress in channel modeling for mmWave [17]–[20], the investigation on blockage modeling has been limited. In [18], a Boolean scheme has been used to model the blockage due to buildings in urban cellular network. However, the main limitation of this work is that only the direct propagation is considered and the reflected signals are ignored. Moreover, it was assumed that each link experiences an independent and identically distributed (i.i.d.) small-scale Rayleigh fading which does not seem to be a good statistical model for mmWave. Another ball-based blocking model has been proposed in [19] to capture, once, again blockages due to building in outdoor environments. Using this model, the coverage probability and the capacity have been derived and compared to real building data obtained from Google Map. In [21], the authors modeled the human bodies as 3D cylinders with fixed heights and diameters. This blockage model has been incorporated in stochastic geometry framework to derive the coverage probability. However, considering fixed heights and diameters does not seem to be reasonable in finite-size geometry.

We note that mmWave communications are expected to operate in shorter distances and in crowded environments [20]. In this case, human bodies can act as blockers to transmitted signals. Clearly, an accurate blockage model must account for the heights of transceivers, their separation, the dimension of blocking objects’ distributions. None of prior works have addressed these issues in their models.

C. Contribution

In this paper, we consider a mmWave SMS consisting of a set of SMs communicating with a gateway. We use the term gateway instead of the receiver as it will send the SMs’ data to the service provider via UMTS, 4G, or WiMax. All nodes are located on the same floor of a building. These nodes may include water, heat and electricity meters. The reason why we assumed that all nodes are located on the same floor, i.e., in the same building is because the interference coming from the smart meters (SMs) located in the basements of neighboring houses, is negligible. In other words, mmWaves can not propagate through building materials like walls because the penetration losses of mmWave are very high. Hence, indoor networks in the same building will be isolated from indoor networks in other buildings due to high penetration losses of mmWave. That is, the results reported in this paper are still valid if other SMs are located in the basement of neighboring houses because the latter will only have a marginal interference effect. In each floor, we may have one gateway or more to receive the data from the nodes belonging to the same floor. Such a communication is subject to interference among the SMs belonging to the same gateway and other SMs belonging to different gateways. Motivated by the work in [22], our channel model includes path loss, and shadowing. It also includes a Nakagami fast fading component which has been justified by [23]. A specific feature of SMS is that the transmitter sensors are kept at their basic functionalities. That is, they are in transmit-only mode, with no channel estimation, no power control, and no awareness of the environment. Obviously, although this feature keeps the most complexity at the receiver side, it increases the probability of collision, thus compromising SMS reliability. To overcome this limitation, our model comprises an Aloha-like medium access protocol, where each SM transmits with a certain probability. At the gateway, a maximum ratio combining (MRC) is used. Note that we considered a fixed network, i.e., the distances of the interferers are fixed because the SMs are attached to the walls of the building. However, the motion of human inside the building can interrupt the signals transmitted from the SMs. Hence, we used stochastic geometry to model the blockages and to study their effect on a fixed mmWave SMS network. For this broad setting, our contributions are as follows:
We derive the outage probability along with the diversity gain of the system, assuming first that the effect of human-body blockage is negligible.

We propose a general and tractable model that incorporates human-body blockages. In this model, humans are represented as cylinders whose centers follow the Poisson point process (PPP) with arbitrary heights and diameters.

Using tools from stochastic geometry, we derived the probability of line of sight (LOS) between each SM and the gateway as function of the separation between the SM and the gateway and their heights.

We derived the probability of outage along with the diversity gain of the system accounting for human-body blockage.

D. Outline

The remainder of the paper is organized as follows: Section II discusses the system model. Section III presents the outage probability without blockages. Section IV presents the outage probability with blockages. Numerical results and interpretations are presented in Section V. Section VI concludes the paper.

II. System Model

The network comprises \( K + 2 \) nodes. It includes a gateway \( R_0 \), a reference SM \( T_0 \), and \( K \) interferers \( T_i \), \( 1 \leq i \leq K \). Each SM has a fixed location because it is attached to the wall inside the building. Each transmitter has a single antenna while the gateway has \( M \) receive antennas as shown in Fig. 1. Our signal model is based on complex baseband model. Although the mmWave spectrum is generally broadband, we assume a flat fading channel since the wideband channel can be split into a set of parallel narrowband channels using orthogonalization techniques [24]. The complex channel coefficients are assumed to be known at the receiver. The signal at each receive antenna is corrupted by an Additive White Gaussian Noise (AWGN) with zero mean and variance \( \sigma^2 \).

We consider separation between the antennas are larger than half-wavelength. Consequently, there is no spatial correlation between receive antennas. During the transmission process and at an instant \( t \), the \( i^{th} \) node transmits the signal \( s_i(t) \) \( \left( \text{we assume } E \left[ s_i(t)^2 \right] = P_i, \text{ the transmitted power of SM } i \right) \). The channel vector between the \( i^{th} \) transmitter and the gateway is equal to \( h_i = [h_{1i}, h_{2i}, ..., h_{Mi}]^T \), where \( h_{ji} \) is the complex channel coefficient between the \( i^{th} \) transmitter and the \( j^{th} \) receive antenna. Following the work in [25], reflections due to the boundaries and walls within the network are implicitly incorporated in the channel gain \( h_i \), \( i = 0, \cdots, K \). In mmWave literature, assuming Nakagami-\( m \) for the channel fading is quite standard [13], [14], [16], [23], [25]. We argue that although some recent measurements in the 73 GHz mmWave band conducted in the outdoor environment in downtown Brooklyn, New York, revealed that the small-scale spatial fading of the received signal voltage amplitude is Rician-distributed [26], the outcome of this measurement campaign is not universal as it may change if different outdoor environments are considered. Therefore, each channel gain \( h_i \) accounts for Nakagami fast fading, path loss, and shadowing as described by:

\[
\begin{align*}
\beta_i, L = &\begin{cases}
\beta_{0,L}, i = 0, \\
\beta_{i,L} \cdot \frac{10}{20} \log_{10} w_i, & 1 \leq i \leq K,
\end{cases}
\end{align*}
\]

where \( \beta_i, L = \beta_{i,L} (d_i) = \left( \frac{d_i}{\bar{d}_{ref}} \right)^{-\alpha_L} \) is the path loss when the transmitter is in line of sight (LOS) which is expressed as a function of \( d_i \) (the distance between the \( i^{th} \) transmitter and the receiver), \( d_{ref} \) is the reference distance, \( \alpha_L \) is the attenuation power-law exponent LOS case, \( \eta_i \) is the shadowing factor and \( w_i = [w_{1i}, w_{2i}, ..., w_{Mi}]^T \) is the fading vector consisting of i.i.d. \( \text{2} \) Nakagami-\( m \) random variables RVs with fading parameter, and spread parameter in LOS \( m_L \) and \( \Omega_L \), respectively. In the presence of Log.Normal shadowing, the \( \{\eta_i\} \) are i.i.d. Gaussian with mean \( \mu_s \), and variance \( \sigma^2_s \).

Without loss of generality, we assume that \( m_L \) is constant over the antenna array. We note that the channel model in (1) has been widely used in the mmWave literature, e.g., [27]. We assume that the path loss and shadowing between the transmitter and each receive antenna are the same. For the multiple access strategy, we suppose an Aloha medium access control (MAC) protocol. The \( i^{th} \) interferer transmits with a probability \( p_i \). We denote by \( I_i \) a Bernoulli RV which has the following probability:

\[
P(I_i) = \begin{cases} 
 p_i, & \text{if } I_i = 1, \\
 1 - p_i, & \text{if } I_i = 0,
\end{cases}
\]

We denote by \( x(t) = [x_1(t), x_2(t), ..., x_M(t)] \) the vector of signals at \( M \) receive antennas.

\[
x(t) = \sum_{i=0}^{K} h_i s_i(t) + n(t),
\]

where \( n(t) \) represents the AWGN vector, with \( n(t) \sim \mathcal{CN}\left(0, \sigma^2 I_M\right) \), a circularly symmetric white Gaussian noise with 0 mean and covariance \( \sigma^2 I_M \).

We assume MRC-based receiver. Therefore, the output signal \( y(t) \) of an M-element antenna array operating in the presence of \( K \) interferers is equal to:

\[
y(t) = L_m^H x(t),
\]

where \( L_m \) is the weight vector for MRC. Taking \( L_m = h_0 \) leads to the following equation

\[
y(t) = h_0^H h_0 s_0(t) + \sum_{i=1}^{K} h_i^H h_i s_i(t) + h_0^H n(t).
\]

1Although \( \beta_i, L (d_i) \) depends on \( d_i \), we will write it simply as \( \beta_i, L \) for convenience.

2Note that our derivation still holds when these components are independent and non-identically distributed. However, the analytical results will be more complicated.
III. OUTAGE PERFORMANCE

From (5), it is clear that decoding interference as noise makes the signal-to-interference-plus-noise-ratio (SNIR) as the yardstick of interest for analyzing the outage probability of SSM. Thus, the SNIR can be computed as:

$$\gamma(\nu_0, \nu) = \frac{P_0(h_i^H h_0)^2}{\sum_{i=1}^{K} P_i (h_i^H h_i + h_0^H h_0) + \sigma^2(h_0^H h_0)}$$

$$= \frac{P_0 \zeta_0}{\sum_{i=1}^{K} P_i \zeta_i + \sigma^2 \zeta_0}$$

where $\zeta_i = \beta_i, 10^{\frac{b_i}{10}}, b_i = \frac{\nu_i m_i^L}{\nu_0 m_0^L}, \nu_0 = \frac{P_0}{\sigma^2}$, and $\nu = [\nu_1 = \frac{P_0}{\nu_0}], \ldots, \nu_K = \frac{P_k}{\nu_0}$]. In (6), $\zeta_i$ represents the instantaneous channel between the $i^{th}$ SM and the gateway, $\nu_0$ represents the transmit SNR of the SM of interest, and $\nu_i, 1 \leq i \leq K$, represents the transmit SNR at each interfering SM. The outage probability is defined as the probability that the received SNIR is below a given threshold $\gamma_T$ for a given $\nu_0$ and $\nu$, i.e.,

$$P_{out}(\gamma_T, \nu_0, \nu) = \Pr(\gamma(\nu_0, \nu) \leq \gamma_T).$$

In this subsection, we derive the conditioned outage probability $P_{out}(\gamma_T, \nu_0, \nu|\chi)$ for a given shadowing factor vector $\chi = [\chi_0 = 10^{\frac{b_0}{10}}, \ldots, \chi_K = 10^{\frac{b_K}{10}}]$.

**Theorem 1.** The conditioned outage probability of SSM can be expressed as follows:

$$P_{out}(\gamma_T, \nu_0, \nu|\chi) = \sum_{j=0}^{\infty} \frac{(-1)^j T_0^{m_i L + j} \Gamma(m_i L + j)}{\sum_{S \subseteq E} \left( \prod_{i \in S} (1 - p_i) \prod_{i \notin S} p_i \right) \prod_{i \in S} \left( M S m_i L \sum_{k=0}^{\infty} \prod_{i \in S} \left( \frac{\psi_k}{\psi_{\min}} \right) \prod_{i \notin S} \left( \frac{\psi_{\min}}{\psi_k} \right) \right) \prod_{k=0}^{m_i L + j} \frac{\delta_k}{\Gamma(N S m_i L + k)} \Gamma(-m_i L - j)}$$

where the second sum in (8) is overall non empty subsets $S$ in the set $E = \{1, \ldots, K\}, N_S = \text{card}(S)$ is the cardinal of $S, \psi_i = \frac{\psi_{\min}}{\psi_{\max}}, \psi_{\min} = \min(\psi_i), \psi_{\max} = \max(\psi_i), L_i, \psi_k = \beta_i, L_i, \Omega_L$ the average channel gain of the $i^{th}$ SM conditioned on $\chi_i$, $T_0 = \frac{m_i L}{\nu_0}, \Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$ is the Gamma function, $G_{m,n}^{p,q}(a_1, \ldots, a_p, b_1, \ldots, b_q, z)$ is the Meijer $G$-function, and the coefficients $\delta_k$ can be obtained recursively using the formula:

$$\begin{cases} \delta_0 = 1, \\ \delta_{k+1} = \frac{1}{k+1} \sum_{r=1}^{k+1} \left( \sum_{i \in S} m_i L \left( 1 - \frac{\psi_{\min}}{\psi_i} \right)^r \right) \delta_{k+1-r}, \quad k = 0, 1, 2, \ldots \end{cases}$$

**Proof:** The proof is presented in Appendix A. $\blacksquare$

We note that the results in Theorem 1 provide an analytical closed-form of the outage probability conditioned on $\chi$. Below we provide insights into how Theorem 1 could be used to devise performance of our communication system in various particular cases. For example, by setting $p_i = 0, \forall i \in \{1, \ldots, K\}$. Theorem 1 captures a time division multiple access (TDMA) scheme when only one transmitter is allowed to communicate at time. In this case, the conditioned outage probability in (8) simplifies to:

$$P_{out}(\gamma_T, \nu_0, \nu|\chi) = 1 - \frac{\Gamma(m_i L, T_0)}{\Gamma(m_i L)}$$

where $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete Gamma function. On the other extreme, Theorem 1 also captures the case of fully loaded system when all the transmitters are communicating at the same time, by setting $p_i = 1, \forall i \in \{1, \ldots, K\}$. In absence of shadowing and if we assume a receiver with one single antennas, we obtain the same result as in [25]. However, when $m_i L$ is not an integer, it is difficult to find a closed-form expression of the (unconditioned) outage probability $P_{out}(\gamma_T, \nu_0, \nu)$ by taking the expectation with respect to $\chi$. Nevertheless, it can be accurately estimated through Monte Carlo simulations. However, a closed-form expression of $P_{out}(\gamma_T, \nu_0, \nu)$ may be obtained when $m_i L$ is integer. This is formalized below.

**Theorem 2.** When $m_i L$ is integer, the outage probability of SSM can be expressed by:

$$P_{out}(\gamma_T, \nu_0, \nu|\chi) = \frac{1}{\Gamma(m_i L)} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(m_i L + k)}{\nu_0} \left( \frac{\gamma_T m_i L}{\nu_0} \right)^k \Gamma(m_i L + k)$$

$$= \sum_{t=0}^{\infty} \left( \frac{\gamma_T m_i L}{t} \right)^t \sum_{i=1}^{K} \prod_{i=1}^{t} \left( 1 - p_i \right) \delta_i$$

$$+ \frac{p_i \Gamma(t_i + m_i L)}{t_i \Gamma(m_i L)} \left( \frac{\nu_i}{\nu_0} \right)^{t_i} \Gamma(m_i L + t_i),$$

where $\delta_i$ is the Kronecker delta function defined as:

$$\delta_i = \begin{cases} 1, & \text{if } t_i = 0, \\ 0, & \text{if } t_i = 1, \end{cases}$$

and $\lambda_{i, L} = E_{\chi_i}(\chi_{i, L}) = \beta_i, L_i, \Omega_L E_{\chi_i}(\chi_i) = \beta_i, L_i, \Omega_L e^{\frac{t_0}{\sigma^2}}, \psi_k = \text{the average channel gain of the } i^{th} \text{ SM}, f_{\sigma^2, +}(x) = e^{\frac{1}{2} \sigma^2} x(x+1)$, and $f_{\sigma^2, -}(x) = e^{\frac{1}{2} \sigma^2} x(x-1)$.

**Proof:** The proof is presented in Appendix B. $\blacksquare$
SM of interest, and derive an asymptotic expression for the outage probability, which enables the characterization of the achievable diversity order. Specifically, we characterize the two key performance parameters dictating the outage probability in the high SNR regime, i.e., the diversity gain $G_d$ and the array gain $G_a$ defined by [22]:

$$P_{out}^{\infty}(\gamma_T, \nu_0, \nu) = G_a(\gamma_T, \nu) \nu_0^{-G_d} + O\left(\nu_0^{-(G_d+1)}\right), \quad (13)$$

where $O\left(\nu_0^{-(G_d+1)}\right)$ is a function of $\nu_0$ such that $O\left(\frac{\nu_0}{\nu_0^{-(G_d+1)}}\right) \leq M_0$, for some positive $M_0$.

**Corollary 1.** In the high SNR regime, i.e., $\nu_0 \to \infty$, the outage probability of the system can be expressed as:

$$P_{out}^{\infty}(\gamma_T, \nu_0, \nu) = G_a(\gamma_T, \nu) \nu_0^{-G_d} + O\left(\nu_0^{-(G_d+1)}\right), \quad (14)$$

where

$$G_d = m_L M, \quad (15)$$

and $E_{\chi}(g(\chi))$ denotes the expectation of $g(\chi)$ with respect to $\chi$.

**Proof:** The proof is presented in Appendix C.

Corollary 1 presents the asymptotic expression for the outage probability. It indicates that SMS achieves a diversity gain of $M_{mL}$. It depends only on the number of receive antennas at the gateway and the Nakagami-fading parameter of the reference transmitter. By increasing one of those parameters, we increase the diversity gain and thus ameliorate the reliability of the system. Note that $G_a(\gamma_T, \nu) > 0$ since the last product in (16) is the probability density function (PDF) of sum of gamma variate.

**IV. OUTAGE PERFORMANCE WITH BLOCKAGE**

In this section, we study the effect of the blockage on the performance of our system. The potential sources of blockage for mmWave in indoor environments are humans and/or concrete structures. We model them by cylinders with a certain height, $H$, and the base diameter $D$ [29]. Differently from [18], [19], [21] that considered cubes-based blocking model with a fixed height, ball-based blocking model with a fixed radius, or 3D cylinders with fixed heights and diameters, we consider in our model both $H$ and $D$ are RVs. We model human bodies by cylinders with different sizes. The distribution of height is a Truncated Normal with mean $\mu_H$ and variance $\sigma_H^2$ and lies within the interval $[0, H_a]$, where $H_a$ is the height of the ceiling as illustrated in Fig. 2 [30]. The RV $D$ is assumed to be uniformly-distributed between $d_{\min}$ and $d_{\max}$. We use stochastic geometry to model these blockages. As $D$ is a RV, the centers of cylinders bases follow a Matern hard-core point process (MHCP) of type I [31], [32] on the plane with the intensity $\lambda_b$ [33]. This process is appealing to ensure that the locations of blockers do not overlap. However, due to its complexity, MHCP is commonly replaced by the more tractable PPP with intensity $\lambda_0 \geq \lambda_b$ [33]. In this section, we consider that each SM is located at certain height $H_i$ above the ground, and the gateway located at the height $H_G$ as illustrated in Fig. 2 with $H_i$ and $H_G \in [0, H_a]$. As a practical assumption, all $H_i$ are smaller than $H_G$, i.e., $H_i \leq H_G$, $i \in \{0, \cdots, K\}$. Our objective is to derive the system outage probability. As the latter depends on whether the SM is in LOS or in NLOS with the gateway, the attenuation power-law exponent and the fading parameter of the $i^{th}$ SM have two possible values as follows

$$\begin{cases} \alpha_L \text{ w.p. } P_{LOS}(d_i), & \text{if } m_L \text{ w.p. } P_{LOS}(d_i), \\ \alpha_N \text{ w.p. } 1 - P_{LOS}(d_i), & \text{if } m_N \text{ w.p. } 1 - P_{LOS}(d_i), \end{cases} \quad (17)$$

where $\alpha_L$ is the attenuation power-law exponent for the LOS and $\alpha_N$ for NLOS, $m_L$ is the Nakagami factor for the LOS, and $m_N$ for NLOS, respectively. Following the work in [25], reflections from human bodies are implicitly incorporated in LOS and NLOS attenuation power-law exponents, i.e., $\alpha_L$ and $\alpha_N$, respectively, which ideally would be determined based on ray tracing or measurement results. We first evaluate the probability of LOS below.

**Lemma 1.** When the network region is rectangular and blockages have diameter $D \sim U[d_{\min}, d_{\max}]$ with height having Truncated Normal distribution with mean $\mu_H$ and variance $\sigma_H^2$ and lies within the interval $[0, H_a]$, the probability that the $i^{th}$ SM at distance $d_i$ from the gateway is in LOS is given by (18).

**Proof:** For convenience, the proof is presented in Ap-
\[ P_{\text{LOS}}(d_i) = \begin{cases} 
\exp \left( -\frac{1}{2} \left( Q(d_{\text{max}}, d_{\text{min}}) \lambda_b d_i \left( \frac{n_H - \mu H}{\sigma_H^2} \right)^2 \right) \right) & \text{for } H_R \neq H_i, \\
\exp \left( -\frac{1}{2} \left( Q(d_{\text{max}}, d_{\text{min}}) \lambda_b d_i \left( \frac{n_H - \mu H}{\sigma_H^2} \right)^2 \right) \right) & \text{otherwise}, 
\end{cases} \]

where \( Q(d_{\text{max}}, d_{\text{min}}) = d_{\text{min}} + \frac{d_{\text{max}} + (d_{\text{min}} - d_{\text{max}})\lambda_b}{(d_{\text{max}} - d_{\text{min}})\lambda_b} \), \( \Phi(x) = \frac{1}{2} \left[ 1 + \text{erf}(x) \right] \), \( \text{erf}(x) \) is the exponential function, and \( \text{erf}(\cdot) \) is the error function.


Corollary 2. In the presence of blockages, the outage probability of the SMS is equal to:

\[ P_{\text{out}}(\gamma_T, \nu_0, \nu) = P_{\text{LOS}}(d_0) P_{\text{out}, L}(\gamma_T, \nu_0, \nu) + (1 - P_{\text{LOS}}(d_0)) P_{\text{out}, N}(\gamma_T, \nu_0, \nu), \]

where \( P_{\text{out}, L}(\gamma_T, \nu_0, \nu) \) and \( P_{\text{out}, N}(\gamma_T, \nu_0, \nu) \) are the outage probabilities of SM of interest when it is in LOS or NLOS with the gateway, respectively, given by:

\[ P_{\text{out}, L}(\gamma_T, \nu_0, \nu) = \frac{1}{\Gamma(m_L M_L)} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_0 L} \right)^{M m_L} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(M m_L + k)} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_0 L} \right)^k f_{\sigma_+, +}(M m_L + k) \times \sum_{t=0}^{M m_L + k} \left( \begin{array}{c} M m_L + k \\ t \end{array} \right) t! \sum_{i=1}^K \prod_{i=1}^K \left[ (1 - p_i) \delta_{t_i} + p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L) \left( \frac{\nu_i \lambda_i L}{m_L} \right)^{t_i} f_{\sigma_+, -}(t_i) \right] \]

\[ P_{\text{out}, N}(\gamma_T, \nu_0, \nu) = \frac{1}{\Gamma(m_N M_N)} \left( \frac{\gamma_T m_N}{\nu_0 \lambda_0 N} \right)^{M m_N} \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(M m_N + k)} \left( \frac{\gamma_T m_N}{\nu_0 \lambda_0 N} \right)^k f_{\sigma_+, +}(M m_N + k) \times \sum_{t=0}^{M m_N + k} \left( \begin{array}{c} M m_N + k \\ t \end{array} \right) t! \sum_{i=1}^K \prod_{i=1}^K \left[ (1 - p_i) \delta_{t_i} + p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L) \left( \frac{\nu_i \lambda_i L}{m_L} \right)^{t_i} f_{\sigma_+, -}(t_i) \right]. \]

Next, we study the high SNR regime of the SM of interest, and derive an asymptotic expression for the outage probability in the presence of blockages, which enables the characterization of the achievable diversity order.

Corollary 3. In the high SNR regime, the outage probability of the system, in presence of blockages, can be expressed by:

\[ P_{\text{out}}^\infty(\gamma_T, \nu_0, \nu) = (1 - P_{\text{LOS}}(d_0)) P_{\text{out}, N}(\gamma_T, \nu_0, \nu) + O(\nu_0^{-(m_N M_N + 1)}), \]

where

\[ P_{\text{out}, N}(\gamma_T, \nu_0, \nu) \approx \frac{\gamma_T m_N}{\nu_0 \lambda_0 N} \frac{M m_N}{M_N M} \frac{f_{\sigma_+, +}(M m_N)}{\Gamma(m_N M + 1)} \times \sum_{t=0}^{M m_N} \left( \begin{array}{c} M m_N \\ t \end{array} \right) t! \sum_{i=1}^K \prod_{i=1}^K \left[ (1 - p_i) \delta_{t_i} + p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L) \left( \frac{\nu_i \lambda_i L}{m_L} \right)^{t_i} f_{\sigma_+, -}(t_i) \right]. \]

Proof: The proof is presented in Appendix F.

Note that, in the high average SNR regime, \( P_{\text{out}, L}(\gamma_T, \nu_0, \nu) \ll P_{\text{out}, N}(\gamma_T, \nu_0, \nu) \). In this case, the achievable diversity order is equal to \( M m_N \), as indicated by (23).

V. NUMERICAL RESULTS

A. Numerical results for SMS without blockages

In this section, we present numerical results for the outage probability in the absence of blockages. Therefore, all SMs
are in LOS with the gateway. The SMs are located in a uniform $5 \times 9$ rectangular grid as shown in Fig. 3. The number of interferers $K = 43$. The SM of interest is located at distance $d_0 = 5$ m from the gateway. The distances of the 43 interferers to the gateway are given in Fig. 3. We choose the distances between the gateway and SMs so small because they are confined to the indoor environment, i.e., the gateway and the SMs are located in the same floor of the building. The latter is located at the centre of the rectangle. The parameters used in this section are summarized in Table I. Since the conditioned outage probability (8) is complicated, it is difficult to study analytically its convergence and truncation. To solve this problem, we propose numerical results. In our simulation results, we plot in the same figure, the true conditioned outage probability $P_{\text{out}}(\gamma_T, \nu_0, \nu | \chi)$ using Monte Carlo simulations and the approximated one $P_{\text{out}}^{\text{app}}(\gamma_T, \nu_0, \nu | \chi)$ which is given by

$$P_{\text{out}}^{\text{app}}(\gamma_T, \nu_0, \nu | \chi) = \sum_{j=0}^{L_1} \frac{(-1)^j T_0^{m_L M+j}}{j! (m_L M + j)} \Gamma (m_L M) \times \left[ \prod_{i=1}^{K} (1 - p_i) + \sum_{S \in E} \left[ \prod_{i \in S, i \in E_i} p_i (1 - p_i) \right] \right] \left[ \prod_{i \in S} \left( \frac{\psi_{\min}}{\psi_i} \right)^{m_L L_2} \sum_{k=0}^{\delta_k} \Gamma (N_S m_L + k) \Gamma (-m_L M - j) \right] \times G_{2,1}^{1,2} \left[ -N_S m_L - k + 1, m_L M + j + 1 \left| \psi_{\min} \right. \right],$$

(24)

where $L_1$ and $L_2$ denote the order of truncation of the first and the second infinite summation, respectively. We aim to analyze the accuracy of the Truncated outage probability (24) for different values of the Truncated orders. To this end, Fig. 4 shows the variation of $P_{\text{out}}^{\text{app}}(\gamma_T, \nu_0, \nu | \chi)$ as a function of $\gamma_T$ when $L_1 = 1, 2, 5, 8$ and $L_2 = 2, 6, 10$. This figure shows that the high accuracy of (24) in retrieving the values of the true conditioned outage probability is achieved when $L_2 = 10$ (Fig. 4.c). In particular, it is important to note that when $(L_1, L_2) = (5, 10)$, (24) achieves 98% of the true $P_{\text{out}}(\gamma_T, \nu_0, \nu | \chi)$. When $(L_1, L_2) = (8, 10)$, our analytical formula coincides perfectly with the simulation. Moreover, the smaller the value of $\gamma_T$ is the less is the number of truncation order $L_1$ needed to guarantee a fixed accuracy requirement. Now, we study the complexity analysis of the conditioned outage probability (24). The number of different operations required by (24), i.e., the additions and the multiplications, is listed in Table II. The number of additions required by (24) scales linearly with $L_1$, and $L_2$. However, the number of multiplications required by (24) scales quadratically with $L_1$ and linearly with $L_2$. With the above complexity along with

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>5 m</td>
<td>Distance between the gateway and the SM of interest</td>
</tr>
<tr>
<td>$d_{ref}$</td>
<td>1 m</td>
<td>The reference distance</td>
</tr>
<tr>
<td>$m_L$</td>
<td>4</td>
<td>Nakagami fading parameter for LOS link</td>
</tr>
<tr>
<td>$m_N$</td>
<td>2</td>
<td>Nakagami fading parameter for NLOS link</td>
</tr>
<tr>
<td>$\Omega_L = \Omega_N$</td>
<td>1</td>
<td>Nakagami spread parameters for LOS and NLOS link are equal to one</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>2</td>
<td>Path-loss exponent for LOS link</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>4</td>
<td>Path-loss exponent for NLOS link</td>
</tr>
<tr>
<td>$K$</td>
<td>43</td>
<td>Number of interfering SM</td>
</tr>
<tr>
<td>$p_i$, $1 \leq i \leq K$</td>
<td>$p_i$</td>
<td>All the interfering SMs transmit with the same probability of success $p_i$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0 dB</td>
<td>The mean of shadowing factor $\eta_i \sim N(\mu, \sigma)$</td>
</tr>
<tr>
<td>$H$</td>
<td>$N (1.7 \text{m}, 0.1 \text{m})$</td>
<td>Height of blocker, Truncated normal distribution $N(\mu_H, \sigma_H)$ within the interval $[0, H_z]$</td>
</tr>
<tr>
<td>$D$</td>
<td>$U (0.2 \text{m}, 0.8 \text{m})$</td>
<td>Diameter of blocker. $U(d_{min}, d_{max})$</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.3 blockers/m$^2$</td>
<td>Intensity of blockers</td>
</tr>
<tr>
<td>$H_i$, $0 \leq i \leq K$</td>
<td>1.3 m</td>
<td>All the SMs have the same height</td>
</tr>
<tr>
<td>$H_R$</td>
<td>4 m</td>
<td>The height of the gateway</td>
</tr>
<tr>
<td>$H_s$</td>
<td>6 m</td>
<td>The height of the ceiling</td>
</tr>
</tbody>
</table>

Table I: Simulation Parameters

Figure 3: The locations of SMs in a uniform rectangular grid of size $5 \times 9$. 
Table II: Complexity analysis of the approximated conditioned constants which depend on the system parameters.

| $P_{out}^{pp}(\gamma_T,\nu_0,\nu|\chi)$ | Multiplications | Additions |
|---------------------------------|----------------|-----------|
| $2L_1^2 + 2L_1L_2 + (L_2 + 1)C_3$ | $2L_1 + 6L_2$ |           |

Figure 4: Conditioned outage probability vs SINR threshold $\gamma_T$ for different truncation orders values $L_1$ and $L_2$ ($\nu_i = 20$ dB, $i \in \{0, \ldots, K\}$, $M = 1$, $\sigma_n^2 = 10$ dB, and $p_t = 0.1$).

The numerical investigation in Fig. 4, it is clear that truncation of $P_{out}(\gamma_T,\nu_0,\nu|\chi)$ as per (24) is efficient and has only a few number of terms.

Figure 5 shows the outage probability as a function of the transmit SNR $\nu_0$ of the reference SM. This figure illustrates that Monte Carlo simulations matches the analytical expression in (11). The asymptotic expression given in (14) provides an accurate prediction of the outage probability in the high SNR regime. It can be seen from this figure, that increasing the number of branches at the receiver, improves the reliability of the system. This can be explained by the fact that the diversity order ($G_d = Mm_L$) depends on the number of diversity branches.

The dependence of $P_{out}(\gamma_T,\nu_0,\nu)$ on different transmission probabilities $p_t$ is shown in Fig. 6. We clearly observe that the outage probability increases with $p_t$, for a given threshold.

Next, we study the effect of the number of interferers on the performance of the system in 900 MHz (RF band) and 60 GHz (mmWave band) in indoor environment. We follow along the same lines as in [34] to obtain the empirical coefficient values of $\sigma_n^2$ and $\alpha_L$ for the same indoor environment (office) at 60 GHz and 900 MHz, respectively. In Fig. 7, when there is no interfering signals, i.e., a TDMA scheme, the performance becomes better. As the number of interferers increases, the performance gets worse. Figure 7 also shows that mmWave improves the reliability substantially compared to the RF band considered (900 MHz) in the low SINR threshold $\gamma_T$ ($-10$ dB $< \gamma_T < 4$ dB in the figure), for $K = 43$. When $\gamma_T > 4$ dB, the figure, lower carrier frequency (900 Mhz) is advantageous than mmWave frequency (60 GHz) for $K = 43$. This is because of the high variation of the received SINR $\gamma(\nu_0,\nu)$ of lower carrier frequency (900 Mhz) compared to mmWave frequency (60 Ghz), i.e., $\sigma_n^2 = 9.6$ dB $> \sigma_n^2 = 3.92$ dB. It is noteworthy mentioning that the TDMA performance depicted in Fig. 7 is just a benchmark that would never been achievable in realistic scenarios. This is because, interference are generated from sensors belonging to both the gateway of interest, and other adjacent gateways.

Figure 8 illustrates the variation of the outage probability with the distance between the SM of interest and the gateway $d_0$. As can be seen in Fig. 8, the transmission at 60 GHz is beneficial in terms of communication’s reliability when $d_0 \leq 6$ m. However, when $d_0 \geq 6$ m, the lower carrier frequency considered improves the reliability substantially compared to the mmWave band. This confirms that mmWave communications are expected to operate in shorter distances.

Figure 9 illustrates the variations of the outage probability
Outage probability 

The outage probability decreases rapidly as the SINR threshold increases. For instance, as the SINR threshold increases from 20 dB to 25 dB, the outage probability decreases by 80%, as shown in Fig. 7.

Figure 7: Outage probability vs SINR threshold $\gamma_T$ for different transmission probability values $p_t$ ($\nu = 20$ dB, $i \in \{0, \ldots, K\}$, $M = 1$, and $\alpha_s^2 = 0$ dB).

Figure 8: Outage probability vs distance between the SM of interest and the gateway $d_0$ in 900 MHz (RF band) and 60 GHz (mmWave band) ($\nu_t = 20$ dB, $i \in \{0, \ldots, K\}$, $M = 1$, $\sigma_s^2 = 3.92$ dB and $\alpha_L = 2.2$ for 60 GHz, $\sigma_s^2 = 9.6$ dB and $\alpha_L = 2.4$ for 900 MHz, $\gamma_T = 0$ dB, and $p_t = 0.1$).

Figure 9: Outage probability vs distance between the SM of interest and the gateway $d_0$ in 900 MHz (RF band) and 60 GHz (mmWave band) ($\nu_t = 20$ dB, $i \in \{0, \ldots, K\}$, $M = 1$, $\sigma_s^2 = 0$ dB, $\gamma_T = 0$ dB, and $p_t = 0.1$).

B. Numerical results for SMS with blockages

The coverage probability, i.e., $P_c(\gamma_T, \nu_0, \nu) = 1 - P_{\text{out}}(\gamma_T, \nu_0, \nu)$, is plotted versus the threshold $\gamma_T$, for $K = 4$ and $K = 44$, with and without blockages in Fig. 10. An interesting result is that the blockage may improve the coverage at finite SNR, if the number of interferers is low ($K = 4$ in Fig. 10). This result can be explained by the fact that blockages are more likely to attenuate the effect of the interferers. This is a good feature of mmWave frequencies over lower frequencies as the former are much more sensitive to blockages than the latter. As illustrated in Fig. 10, when the coverage probability equals to 0.8, the presence of blockages with 4 interferers provides a 3 dB gain over the absence of blockages. Another observation is that when the number of interferers is very large, the coverage probability converges to the case with no

with the distance between the SM of interest and the gateway $d_0$ for different transmit SNR $\nu_0$. As can be seen in Fig. 8, as $d_0$ increases, the outage probability increases because of the high path loss which degrades the channel quality between the smart meter of interest and the gateway. In addition, as $\nu_0$ increases, the outage probability drops rapidly. For instance, when $d_0 = 10$ m, transmitting with $\nu_0 = 30$ dB drops the outage probability by 80% compared to transmitting with $\nu_0 = 20$ dB.
blockages, i.e., increasing $K$ will close the gap. This can be explained by the fact that increasing the number of interferers with a fixed blockage density is equivalent to decreasing the blockage density. Figure 11 shows the variation of the coverage probability in function with $\lambda_b$, the blockage density. It is seen that as $\lambda_b$ increases, the coverage probability improves rapidly in the case when $\gamma_T = 4$ dB. This supports our interpretation of Fig. 10 that a high density of blockages improves the probability of coverage. Figure 12 illustrates the variation of the outage probability with the SINR threshold $\gamma_T$ for different values of fading parameter of LOS link $m_L$ and with a fixed value of fading parameter of NLOS link $m_N$, i.e., $(m_L = 4, m_N = 3), (m_L = 5, m_N = 3)$ and $(m_L = 6, m_N = 3)$. Figure 12 shows that by increasing $m_L$ in the low SINR threshold regime, ($-10$ dB $< \gamma_T < 1.7$ dB in the figure), the reliability of the system improves. This is because the impact of the interference is negligible in the low SINR regime. As a result, increasing $m_L$ implies that the SM of interest has higher probability to be in LOS with the gateway. However, in the high SINR threshold regime ($1.7$ dB $< \gamma_T < 10$ dB in the figure), the performance of the system deteriorates in terms of reliability by increasing $m_L$. This is because the effect of the interference in the high SINR threshold regime dominates that of the noise, and increasing $m_L$ implies that the interferers have higher probability to be in LOS with the gateway (This is an interference-limited regime).

Figure 13 illustrates the variation of the outage probability with the SINR threshold $\gamma_T$ for different values of fading parameter of NLOS link $m_N$ and with a fixed value of
fading parameter of LOS link $m_L$, i.e., $(m_L = 5, m_N = 2), (m_L = 5, m_N = 3)$ and $(m_L = 5, m_N = 4)$. Figure 13 shows that by increasing $m_N$ in the low SINR threshold regime, ($-10$ dB $< \gamma_T < -2$ dB in the figure), the reliability of the system enhances. This is because the interference is negligible in the low SINR threshold regime (This is a noise-limited regime). As a result, increasing $m_N$ improves the NLOS component of the SM of interest. However, in the high SINR threshold regime, ($-2$ dB $< \gamma_T < 10$ dB in the figure), the performance of the system degrades in terms of reliability by increasing $m_N$. This is because the effect of the interference in the high SINR threshold regime dominates that of the noise, and increasing $m_N$ improves the NLOS components of the interferers (This is an interference-limited regime).

VI. CONCLUSION

In this paper, we have studied the outage probability of mmWave SMS operating in an indoor environment. We incorporated path-loss, shadowing, and Nakagami fading into our channel. We assumed Aloha medium-access protocol and we proposed a framework to model random blockages in an indoor environment using concepts and tools from stochastic geometry. The key idea is to model humans as random cylinders whose centers follow PPP with arbitrary heights, and diameters. Based on this geometric blockage model, we derived the probability of outage, and the corresponding diversity, and array gains. The model captures the dependence of probability of LOS on the distances. Our framework also highlights the importance way in the sense that it may improve the coverage outage probability. Perhaps surprisingly, the developed analysis indicates that the blockages change the behavior of SMS in an important way in the sense that it may improve the coverage probability at finite SNR.

In this paper, we assumed that the receiver treats the interference as a noise. It would be interesting to consider an interference cancellation technique like Successive Interference Cancellation (SIC) in future work. Using an interference mitigation technique would enable SMs to communicate with the gateway simultaneously in a non-orthogonal way and thus achieve a higher data throughput. Furthermore, it is also an interesting topic to extend the blockage model to incorporate non-convex blockages as human bodies are non-convex in shapes.

APPENDIX A

PROOF OF THEOREM 1

We define the conditioned outage probability $P_{\text{out}}(\gamma_T, \nu_0, \nu | \chi)$ as the probability that the received SINR is below a given threshold $\gamma_T$ for a given $\nu_0$, and $\nu = [\nu_1, \ldots, \nu_K]$ conditioned on $\chi = |\chi_0 = 10^{\nu_0/10}, \ldots, \chi_K = 10^{\nu_K/10}|$. Therefore, we can write

$$P_{\text{out}}(\gamma_T, \nu_0, \nu | \chi) = \Pr\left(\gamma(\nu_0, \nu) = \frac{\nu_0 \gamma_0}{\sum_{i=1}^{K} I_i^2 \nu_i \zeta_i + 1} \leq \gamma_T | \chi\right).$$ (25)

We define $R \triangleq \gamma_T^{-1} \nu_0 \gamma_0$, and $Z \triangleq \frac{\nu_0}{\sum_{i=1}^{K} I_i^2 \nu_i \zeta_i}$. The conditioned outage probability can be expressed as follows:

$$P_{\text{out}}(\gamma_T, \nu_0, \nu | \chi) = 1 - \Pr\left(R \geq Z + 1 \bigg| \chi\right) \nonumber \triangleq 1 - P(\gamma_T, \nu_0, \nu | \chi).$$ (26)

Conditioned on $\chi$, let $f_Z(z)$ denotes PDF of $Z$ and $f_R(r)$ denotes the PDF of $R$. Using these definitions, $P(\gamma_T, \nu_0, \nu | \chi)$ may be expressed as:

$$P(\gamma_T, \nu_0, \nu | \chi) = \int_{0}^{\infty} f_Z(z) \left( \int_{(z+1)}^{\infty} f_R(r) dr \right) dz.$$ (27)

When the transmission is made over Nakagami-m fading channels, it is possible to show [35] that $b_i$ ($1 \leq i \leq K$), and $b_0$ follow a Gamma distribution with a shape, and scale parameters $(m_L, \frac{\nu_i}{m_L}), (m_L, \frac{\nu_0}{m_L}),$ respectively. Therefore, $\zeta_i$ ($1 \leq i \leq K$), and $\zeta_0$ follow a Gamma distribution with a shape, and scale parameters $(m_L, \frac{\nu_i}{m_L}), (m_L, \frac{\nu_0}{m_L}),$ respectively, where $\frac{\nu_i}{m_L} = \beta_{i, L} \chi_i \Omega_i$ is the average channel gain of the $i$th SM conditioned on $\chi_i$ in the LOS case. Hence, $R$ is a Gamma RV with parameters $(m_L, \frac{\nu_0}{m_L})$.

Using the simple transformation $u = \frac{\nu_0}{m_L} r$, the inner integral in (27) becomes:

$$\int_{(z+1)}^{\infty} f_R(r) dr = \frac{\Gamma(m_L, (z + 1) T)}{\Gamma(m_L)},$$ (28)

where $T = \frac{\nu_0}{m_L}$ and $\Gamma(s, x) = \int_{x}^{\infty} t^{s-1} e^{-t} dt$ is the upper incomplete Gamma function.

We have $Z_i = I_i^2 \nu_i \zeta_i$, where $\zeta_i$ is a Gamma RV, and $I_i$ is a Bernoulli RV. Let the RV $N_i = I_i^2$. $N_i$ is also a Bernoulli RV with the same probability of success $p_i$ as $I_i$. So, the PDF of $Z_i$ is:

$$f_{Z_i}(z_i) = (1 - p_i) \delta(z_i) + p_i f_{B_i}(z_i),$$ (29)

where $\delta(z)$ is the delta dirac function, and $f_{B_i}(z_i)$ is the PDF of the Gamma RV $(B_i = \nu_i \zeta_i)$ with shape, and scale parameters, $(m_L)$ and $(\frac{\nu_i}{m_L})$, respectively.

We assume that $\{Z_i\}_{i=1}^{K}$ are independent for a given $\chi$. So, $f_Z(z)$ can be written as $\prod_{i=1}^{K} f_{Z_i}(z_i), \chi$, where the product is the convolution product. After some manipulations, the PDF of $Z$ is given by the following expression:

$$f_Z(z) = \prod_{i=1}^{K} \left(1 - p_i\right) \delta(z) + \sum_{S \subseteq E} \left[ \left( \prod_{i \in S, l \in E \setminus S} p_i (1 - p_i) \right) \times \left( \prod_{i \in S} f_{B_i}(z) \right) \right].$$ (30)
The summation \( \left( \sum_{S \subseteq E} \right) \) is taken over all subsets \( S \) such that \( S \subseteq E = \{1, \ldots, K\} \), and \( \prod_{i \in S} f_{B_i}(z) \) is the convolution product. This product can be seen as the PDF of the sum of Gamma variate. This PDF can be obtained using Moschopoulos Theorem [36], as follows:

\[
\prod_{i \in S} f_{B_i}(z) = \prod_{i \in S} \left( \frac{\psi_{\min}}{\psi_i} \right)^{m_i} \frac{\sum_{k=0}^{\infty} \delta_k e^{-\psi_{\min} m_i} (N_SM_L + k)^{-1}}{\psi_i^{e^{-\psi_{\min} m_i}} (N_SM_L + k)^k},
\]

where \( N_S = \text{card}(S) \) is the cardinal of \( S \), \( \psi_i = \frac{\nu_i \chi_i}{m_i} \) \( \forall i \in S \), \( \psi_{\min} = \min(\psi_i) \), and the coefficients \( \delta_k \) can be obtained recursively using the formula:

\[
\begin{align*}
\delta_0 &= 1, \\
\delta_{k+1} &= \frac{1}{e^{\psi_i}} \sum_{r=1}^{k+1} \left[ \sum_{i \in S} m_i \left( 1 - \frac{\psi_{\min}}{\psi_i} \right)^r \right] \delta_{k+1-r}, \\
& \quad k = 0, 1, 2, \ldots.
\end{align*}
\]

We use [37, Eq. (8.354.2)] to express (28) and after substituting the result in (27), we obtain the following expression:

\[
P(\gamma_T, \nu_0, \nu|\chi) = 1 - \sum_{j} \frac{(-1)^j T^{m_L M + j}}{j! (m_L M + j) \Gamma(m_L M)} \times \int_{0}^{\infty} (z + 1)^{m_L M + j} f_Z(z) \, dz.
\]

Substituting (31) into (30), the inner integral in (33) is equal to:

\[
\int_{0}^{\infty} (z + 1)^{m_L M + j} f_Z(z) \, dz = \frac{K}{\pi} (1 - p_i)
\]

\[
+ \sum_{S \subseteq E} \left[ \prod_{i \in S} \prod_{i \in E \setminus S} p_i (1 - p_i) \right] \times \left( \frac{\psi_{\min}}{\psi_i} \right)^{m_i} \frac{\sum_{k=0}^{\infty} \delta_k e^{-\psi_{\min} m_i} (N_S m_L + k)^{-1}}{\psi_i^{e^{-\psi_{\min} m_i}} (N_S m_L + k)^k},
\]

where \( \sum_{j} \frac{(-1)^j T^{m_L M + j}}{j! (m_L M + j) \Gamma(m_L M)} \times \int_{0}^{\infty} (z + 1)^{m_L M + j} f_Z(z) \, dz = \frac{K}{\pi} (1 - p_i)
\]

\[
+ \sum_{S \subseteq E} \left[ \prod_{i \in S} \prod_{i \in E \setminus S} p_i (1 - p_i) \right] \times \left( \frac{\psi_{\min}}{\psi_i} \right)^{m_i} \frac{\sum_{k=0}^{\infty} \delta_k e^{-\psi_{\min} m_i} (N_S m_L + k)^{-1}}{\psi_i^{e^{-\psi_{\min} m_i}} (N_S m_L + k)^k},
\]

We use [38, Eq. (07.34.03.0271.01)] and [37, Eq. (7.813.1)] to solve the inner integral in (34) as follows:

\[
\int_{0}^{\infty} (z + 1)^{m_L M + j} e^{-\psi_{\min} m_L - k - e^{-\psi_{\min} m_L}} \, dz = \frac{\psi_{\min}^{m_L M + k}}{\Gamma(-m_L M - j)}
\]

\[
\times G_{2,1}^{1,2}\left(-N_S m_L - k, 1 + m_L M + j + 1 \bigg| \psi_{\min} \right).
\]

We substitute (35) into (34), and (34) into (33). In the final step, we substitute (33) into (26) to obtain the final result.
where $\delta_t$ is the Kronecker delta function defined as
\[
\delta_t = \begin{cases} 
1, & \text{if } t_s = 0, \\
0, & \text{if } t_s = 1.
\end{cases}
\] (42)

Substituting (41) into (40), the conditioned outage probability becomes:
\[
P_{\text{out}} (\gamma_T, \nu, \nu | \chi) = \frac{1}{\Gamma (m_L M)} \sum_{k=0}^{\infty} \frac{(-1)^k (2 \gamma_T M)^{M_m L + k}}{k! (M m_L + k)}
\times \sum_{t=0}^{M m_L + k} \binom{M m_L + k}{t} t! \sum_{t_i=1}^{K} G_{(p, i, t, m, m, \nu)} (\lambda_{i, \nu}) \cdot
\] (43)

The next step is to derive the conditioned outage probability $P_{\text{out}} (\gamma_T, \nu, \nu | \chi(0))$ by taking an expectation with respect to $\chi = [\chi_1, ..., \chi_K]$. We assume that $\{\chi_i\}_{i=1}^{K}$ are independent. Therefore, this leads us to evaluate $E_{\chi_{i, L}} [G_{(p, i, t, m, m, \nu)} (\lambda_{i, L})]$.
\[
E_{\chi_{i, L}} [G_{(p, i, t, m, m, \nu)} (\lambda_{i, L})] = \int_{0}^{\infty} f_{\chi_{i, L}} (\lambda_{i, L})
\times G_{(p, i, t, m, m, \nu)} (\lambda_{i, L}) \, d\lambda_{i, L},
\] (44)

where
\[
f_{\chi_{i, L}} (\lambda_{i, L}) = \frac{1}{\lambda_{i, L} \sigma_{b, L} \xi_{i, L} \Omega T} \frac{e^{- (\ln \lambda_{i, L} - \mu_s)^2}}{2\pi}. \] (45)

The PDF of $\chi_{i, L}$ depends on the PDF of the Log.Normal RV $\chi_i = 10^{\frac{Z_i}{2}}$. Using [37, Eq. (3.323.2)], (45) and by making a change of variable $r_i = \ln \lambda_{i, L}$ to calculate (44), we obtain the expression of $P_{\text{out}} (\gamma_T, \nu, \nu | \chi(0))$.

In the final step, we derive $P_{\text{out}} (\gamma_T, \nu, \nu | \chi)$ by taking the expectation with respect to $\chi$, and considering the average channel gain of the $i^{th}$ SM $\lambda_i = E_{\chi_i} (\chi) = \beta_{i, L} \Omega T E_{\chi_i} (\chi_i) = \beta_{i, L} \Omega T e^{\frac{E}{\nu} + \mu_s}$.

APPENDIX C

PROOF OF COROLLARY 1

In the high average SNR regime, $\nu \to \infty$ and thus $T \to 0$. Using the expansion of regularized Gamma function at 0 [38, Eq. (06.06.06.0004.02)], (28) is expressed as follows in the high average SNR:
\[
\frac{\Gamma (m_L M, (z + 1) T)}{\Gamma (m_L M)} = 1 - \frac{(z + 1)^{m_L M} T^{m_L M}}{\Gamma (m_L M + 1)} (1 + O (T^{m_L + 1}))
= 1 - \frac{(z + 1)^{m_L M} T^{m_L M}}{\Gamma (m_L M + 1)} + O (T^{m_L + 1}). \] (46)

By properties of the Gamma function, it is clear that
\[
|f_Z(z) - \frac{\Gamma (m_L M, (z + 1) T)}{\Gamma (m_L M)}| \leq f_Z(z), \forall \zeta \in [0, \infty],
\] (47)

and $f_Z(z) : [0, \infty) \to [0, \infty)$ is integrable as $\int f_Z(z) \, dz = 1$.

Therefore, we can use Lebesgue dominated convergence in (27). Using this Theorem, (46), (30), and taking the expectation with respect to $\chi = [\chi_0 = 10^{\frac{0}{10}}, ..., \chi_K = 10^{\frac{\nu K}{10}}]$, we obtain the formula given in corollary 1.

APPENDIX D

PROOF OF LEMMA 1

Figure 14: Top view of the blocking area.

Figure 15: The interfering SM locating at $d_i$ has a height of $H_i$, while the gateway has a height of $H_G$. Not all blockages which cross OQ blockage the actual propagation path $AB$ in $R^3$, such as blockage $(a)$ in the figure. If a blockage is intersecting $OQ$ at a point $r$ away from the gateway effectively blocks $AB$ if and only if its height is larger than $r$ as blockage $(b)$ in the figure.

The distribution of the number of blockages $L$ is required between each SM and gateway to derive probability of LOS. We show that $L$ is a Poisson distributed RV. We assume that the SMs and the gateway are infinitesimal compared to the blockages (humans). They are represented as points as illustrated in Fig. 14. The centers of blockage are drawn from PPP with parameter $\lambda_0$ inside a rectangular-shaped area with width $W$, and length $L$. Consider a SM $T_i$ located at distance $d_i$ from the gateway. The signal will be blocked if there is a blockage inside the subregion $A$ of the rectangle. This subregion looks like in Fig. 14. The width is bounded by $d_{\text{max}}$, that is, the maximum width of blockers and the length is equal to $d_i$. We call this subregion as the blocking zone.
of the $i^{th}$ SM. From [39] which states that the Poisson Law is preserved by thinning. Therefore, the number of blocker centers follow a Poisson distribution with average equals to $\lambda d_{max} d_i$. However, not all the blockers inside $A$ will block the signals from $T_i$ to the gateway. Hence, we define the following events 

(i) Event $B_1$ states that the blocker’s radius is large enough to cross the LOS between the $T_i$ and $R_0$. 

(ii) Event $C_1$ states that the blocker is high enough to block the LOS. A blockage will block the signal from $T_i$ to $R_0$, if its radius and height are larger and longer enough to cross the LOS. Therefore, the average number of blockage $L$ is equal to: 

$$E(L) = \lambda_0 d_{max} d_i Pr\{B_1\} Pr\{C_1\}.$$ 

(48)

The next step is to calculate the probability of each event. Consider a blockage intersecting the link $OX$ at horizontal distance $r$ away from $X$ which the projection of gateway at the ground. As shown in Fig. 15, the human blocks the direction of propagation path $O'X'$, if its height $H > H_i(r)$, where $h(z)$ can be computed as follows: 

$$H_i(r) = \frac{H_i - H_R}{d_i} r + H_i, \quad r \in [0, d_i].$$ 

(49)

As the number of blockages $L$ inside the blocking zone is Poisson RV, the intersections between the blockages and the link $(OX)$ form a Poisson Process in $(OX)$. Therefore, given that $k$ humans intersect $OX$, the $k$ intersections are independently, and uniformly distributed in the interval $[0, d_i]$ [39, Definition 1.1.1]. Hence, given a blockage intersects $OX$, the conditional probability that it blocks $O'X'$ is: 

$$Pr\{C_1\} = \int_0^{d_i} f_{R_i}(r) Pr[H_i > H_i(r)] dr \int_0^{d_i} f_{R_i}(r) (1 - F_{H_i}(H_i(r))) dr,$$ 

(50)

where $f_{R_i}(r)$ is the PDF of uniform distribution from 0 to $d_i$, and $F_{H_i}(x)$ is the cumulative distribution function (CDF) of the blocker’s height. Hence, we have: 

$$f_{R_i}(r) = \begin{cases} \frac{1}{d_i}, & \text{if } r \in [0, d_i], \\ 0, & \text{otherwise.} \end{cases}$$ 

(51)

Since $H_i$ is a RV with Truncated Normal distribution with mean $\mu_H$ and variance $\sigma_H^2$ and lies within the interval $[0, H_a]$, we have: 

$$F_{H_i}(x) = \Phi\left(\frac{x - \mu_H}{\sigma_H}\right) - \Phi\left(\frac{-\mu_H}{\sigma_H}\right),$$ 

(52)

where $\Phi(x) = \frac{1}{2} \left[1 + erf(x)\right]$ and erf($\cdot$) is the error function.

Substituting (52), and (51) into (50), we obtain: 

$$Pr\{C_1\} = \frac{\Phi\left(\frac{H_a - \mu_H}{\sigma_H}\right) - 2\Phi\left(\frac{-\mu_H}{\sigma_H}\right)}{\Phi\left(\frac{H_a - \mu_H}{\sigma_H}\right) - \Phi\left(\frac{-\mu_H}{\sigma_H}\right)},$$ 

(53)

Using [38, Eqs. (006.25.21.0001.01)], the integral in (53) is expressed as follows: 

$$\int_0^{d_i} \left(\frac{H_i(r) - \mu_H}{\sigma_H \sqrt{2}}\right) dr = \left\{ \begin{array}{ll} g(d_i) - g(0) & \text{for } H_i \neq H_R, \\
\int_0^{d_i} \left(\frac{H_i(r) - \mu_H}{\sigma_H \sqrt{2}}\right) dr & \text{otherwise,} \end{array} \right.$$ 

(54)

where $g_i(x) = \frac{b_i \text{erf}(b_i + a_i x)}{a_i \sqrt{\pi} \sqrt{x}} + x \text{erf}(b_i + a_i x) + e^{-(b_i + a_i x)^2}$. 

Substituting (54) into (53), we obtain the final result as: 

$$Pr\{C_1\} = \left\{ \begin{array}{ll}
\frac{\Phi\left(\frac{H_a - \mu_H}{\sigma_H}\right) - 2\Phi\left(\frac{-\mu_H}{\sigma_H}\right)}{\Phi\left(\frac{H_a - \mu_H}{\sigma_H}\right) - \Phi\left(\frac{-\mu_H}{\sigma_H}\right)}, & \text{for } H_i \neq H_R, \\
\int_0^{d_i} \left(\frac{H_i(r) - \mu_H}{\sigma_H \sqrt{2}}\right) dr & \text{otherwise.} \end{array} \right.$$ 

(55)

The probability of event $B_1$ is the probability that the radius of the blocker’s base is large enough to cross the LOS between $T_i$ and $R$. This means that the absolute value of the blocker’s $y$ coordinate is greater than its radius, i.e. $|y| > \frac{d}{2}$ as illustrated in Fig. 14. As the number of blocker centers in the blocking zone follows a PPP, the $y$ coordinates of each center are independently and uniformly distributed on the interval $[0, d_{max}]$. Hence, the probability of event $B_1$ is equal to: 

$$Pr\{B_1\} = \int_{-d_{max}}^{d_{max}} f_Y(y) Pr[|y| > Rd] dy,$$ 

(56)

where $f_Y(y)$ is the PDF of uniform distribution from $-\frac{d_{max}}{2}$ to $\frac{d_{max}}{2}$, and $F_{Rd}(rd)$ is the CDF of the blocker’s radius. 

Since blocker’s diameter $D$ is uniformly distributed on the interval $[d_{min}, d_{max}]$, its radius $Rd = \frac{D}{2}$ is also uniformly distributed on the interval $[\frac{d_{min}}{2}, \frac{d_{max}}{2}]$ and has the following CDF expression: 

$$F_{Rd}(rd) = \left\{ \begin{array}{ll} 0, & \text{for } rd < \frac{d_{min}}{2}, \\
\frac{2rd - d_{min}}{d_{max} - d_{min}}, & \text{for } rd \in \left[\frac{d_{min}}{2}, \frac{d_{max}}{2}\right), \\
1, & \text{for } rd \geq \frac{d_{max}}{2}. \end{array} \right.$$ 

(57)
$f_Y(y)$ has the following expression:

$$f_Y(y) = \begin{cases} \frac{1}{d_{max}} & \text{for } y \in \left[-\frac{d_{max}}{2}, \frac{d_{max}}{2}\right], \\ 0, & \text{otherwise} \end{cases}$$

(58)

Substituting (58) and (57) into (56), we obtain:

$$\Pr \{B_1\} = \frac{d_{min}}{d_{max}} + \frac{d_{max}^2 + (\frac{d_{min}}{d_{max}} - d_{min}) d_{max}}{d_{max} (d_{max} - d_{min})}. \quad (59)$$

Since $L$ is a Poisson RV with average $E(L) = \lambda_0 d_i \Pr \{B_1\} \Pr \{C_1\}$, the probability that a link with length $d_i$ admits LOS propagation, i.e., no blockages cross the link is:

$$P_{\text{LOS}}(d_i) = \Pr (L = 0) = e^{-E(L)} = e^{-(\lambda_0 d_i \Pr \{B_1\} \Pr \{C_1\})}. \quad (60)$$

Substituting (58) and (55) into (60), we obtain the expression of $P_{\text{LOS}}(d_i)$.

**APPENDIX E**

PROOF OF COROLLARY 2

After defining the probability of LOS of each SM, the fading parameters and the attenuation power-law exponent have two possible values: $m_L$ and $\alpha_L$ for the LOS, and $m_N$ and $\alpha_N$ for the NLOS. For simplicity, we assume that $m_L$ and $m_N$ are integers as the outage probability will be more complicated if they are non integers. Hence, we follow the steps in Theorem 2 to derive the outage probability in the presence of blockage. However, the main difference is that the path loss is now a discrete RV. After considering the probability of transmission of each transmitter ($p_0 = 1$, the SM of interest is always transmitting), the path loss of the $i^{th}$ SM is equal to:

$$0, \quad \text{with probability } 1 - p_i;$$

$$\beta_{i,L} = \left(\frac{d_i}{d_{ref}}\right)^{-\alpha_L}, \quad \text{with probability } p_i P_{\text{LOS}}(d_i);$$

$$\beta_{i,N} = \left(\frac{d_i}{d_{ref}}\right)^{-\alpha_N}, \quad \text{with probability } p_i (1 - P_{\text{LOS}}(d_i)).$$

(61)

Now, the RVs $\{Z_i\}_{i=1}^K$, and $R$ have the following PDF:

$$f_R(r) = P_{\text{LOS}}(d_0) \ f_{C_{0,L}}(r) + (1 - P_{\text{LOS}}(d_0)) \ f_{C_{0,N}}(r), \quad (62)$$

$$f_{Z_i}(z_i) = (1 - p_i) \delta(z_i) + p_i P_{\text{LOS}}(d_i) \ f_{B_{i,L}}(z_i) + p_i (1 - P_{\text{LOS}}(d_i)) \ f_{B_{i,N}}(z_i), \quad (63)$$

where $f_{C_{0,L}}(\cdot)$, $f_{C_{0,N}}(\cdot)$, $f_{B_{i,L}}(\cdot)$, and $f_{B_{i,N}}(\cdot)$ are the PDFs of the Gamma RVs which have the following distributions:

$$C_{0,L} = \text{Gamma} \left(n_M M_m \frac{\lambda_{\gamma L}^0}{m_L}, \frac{\lambda_{\gamma L}^0}{m_L} \right),$$

$$C_{0,N} = \text{Gamma} \left(n_M M_m \frac{\lambda_{\gamma N}^0}{m_N}, \frac{\lambda_{\gamma N}^0}{m_N} \right),$$

$$B_{i,L} = \text{Gamma} \left(m_L, \frac{\lambda_{\gamma L}^0}{m_L} \right),$$

$$B_{i,N} = \text{Gamma} \left(m_N, \frac{\lambda_{\gamma N}^0}{m_N} \right), \quad (64)$$

with $\lambda_{\gamma L}^0 = \beta_{i,L} \lambda_L \Omega_L$ and $\lambda_{\gamma N}^0 = \beta_{i,N} \lambda_N \Omega_N$. Considering these modifications, (28) will have the following expression:

$$\int_{0}^{\infty} f_R(r) dr = P_{\text{LOS}}(d_0) \frac{\Gamma \left(m_L M_m \left(\sum_{i=1}^{K} z_i + 1\right) T_L\right)}{\Gamma \left(m_L M_n\right)} \left(1 - P_{\text{LOS}}(d_0)\right) \frac{\Gamma \left(m_N M_m \left(\sum_{i=1}^{K} z_i + 1\right) T_N\right)}{\Gamma \left(m_N M_n\right)}, \quad (65)$$

where $T_L = \gamma T m_L / (v_0 \lambda_{0,L})$ and $T_N = \gamma T m_N / (v_0 \lambda_{0,N})$. We use the same steps as in Theorem 2 to derive the outage probability in the presence of blockage as in (26), and (28), we added the term which corresponds to case when the SM is in NLOS with the gateway. This addition has no effect on the derivation of the outage probability.

**APPENDIX F**

PROOF OF COROLLARY 3

Starting by the expression of the outage probability in Corollary 2:

$$P_{\text{out}}(\gamma T, v_0, \nu) = P_{\text{LOS}}(d_0) P_{\text{out,L}}(\gamma T, v_0, \nu) \quad (66)$$

In the high SNR regime, $v_0 \to \infty$, $\frac{\gamma T m_L}{v_0 \lambda_{0,L}} \to 0$, and $\frac{\gamma T m_N}{v_0 \lambda_{0,N}} \to 0$. In this case, $P_{\text{out,L}}(\gamma T, v_0, \nu)$ and $P_{\text{out,N}}(\gamma T, v_0, \nu)$ correspond to the first term of each first sum (the first term of the sum corresponds to the index $k = 0$) from $P_{\text{out,L}}(\gamma T, v_0, \nu)$, and $P_{\text{out,N}}(\gamma T, v_0, \nu)$. The outage probability in the high SNR regime is expressed as follows:

$$P_{\text{out}}(\gamma T, v_0, \nu) = P_{\text{LOS}}(d_0) P_{\text{out,L}}(\gamma T, v_0, \nu) \quad (67)$$

where $P_{\text{out,L}}(\gamma T, v_0, \nu)$, and $P_{\text{out,N}}(\gamma T, v_0, \nu)$ are given in Corollary 3. Next, we are going to prove that:

$$P_{\text{LOS}}(d_0) P_{\text{out,L}}(\gamma T, v_0, \nu) \ll (1 - P_{\text{LOS}}(d_0)) \times P_{\text{out,N}}(\gamma T, v_0, \nu), \quad (68)$$

as follows:

$$\lim_{v_0 \to \infty} \frac{P_{\text{LOS}}(d_0) P_{\text{out,L}}(\gamma T, v_0, \nu)}{(1 - P_{\text{LOS}}(d_0)) P_{\text{out,N}}(\gamma T, v_0, \nu)} = \lim_{v_0 \to \infty} \frac{P_{\text{LOS}}(d_0) A_L(\nu)}{(1 - P_{\text{LOS}}(d_0)) A_N(\nu)} \left(\frac{\gamma_T m_L}{v_0 \lambda_{0,L}}\right) \left(\frac{\gamma_T m_N}{v_0 \lambda_{0,N}}\right) \quad (69)$$

$$= \lim_{v_0 \to \infty} \frac{P_{\text{LOS}}(d_0) A_L(\nu)}{(1 - P_{\text{LOS}}(d_0)) A_N(\nu)} \left(\frac{\gamma_T m_L}{\lambda_{0,L}}\right) \left(\frac{\gamma_T m_N}{\lambda_{0,N}}\right) \quad (70)$$

$$\times \left(\frac{\lambda_{0,N}}{\gamma_T \lambda_{0,N}}\right)^{M_{m,L}} \nu_0^{M M_{m,L}} \left(\frac{\lambda_{0,L}}{\gamma_T \lambda_{0,L}}\right)^{M_{m,L}} \nu_0^{M M_{m,L}} \left(\frac{\lambda_{0,N}}{\gamma_T \lambda_{0,N}}\right)^{M_{m,L}} \nu_0^{M M_{m,L}}, \quad (71)$$
where

\[ A_N(\nu) = \frac{f_{\sigma,+}(Mm_N)}{\Gamma(m_NM+1)} \sum_{t=0}^{Mm_N} \binom{Mm_N}{t} t! \]
\times \sum_{i=1}^{K} \prod_{1=i}^{K} \left[ (1-p_i)\delta_{t_i} \right]
+ \frac{p_iP_{\text{LOS}}(d_i)(t_i + m_L)}{t_i!\Gamma(m_L)} \left( \frac{\nu_i\lambda_iL}{m_L} \right)^{t_i} f_{\sigma,-}(t_i)
+ \frac{p_i(1-P_{\text{LOS}}(d_i))(t_i + m_N)}{t_i!\Gamma(m_N)} \left( \frac{\nu_i\lambda_iN}{m_N} \right)^{t_i} f_{\sigma,-}(t_i) \]
\]
\[ A_L(\nu) = \frac{f_{\sigma,+}(Mm_L)}{\Gamma(m_LM+1)} \sum_{t=0}^{Mm_L} \binom{Mm_L}{t} t! \]
\times \sum_{i=1}^{K} \prod_{1=i}^{K} \left[ (1-p_i)\delta_{t_i} \right]
+ \frac{p_iP_{\text{LOS}}(d_i)(t_i + m_L)}{t_i!\Gamma(m_L)} \left( \frac{\nu_i\lambda_iL}{m_L} \right)^{t_i} f_{\sigma,-}(t_i)
+ \frac{p_i(1-P_{\text{LOS}}(d_i))(t_i + m_N)}{t_i!\Gamma(m_N)} \left( \frac{\nu_i\lambda_iN}{m_N} \right)^{t_i} f_{\sigma,-}(t_i) \]
\[ (72) \]
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\[ (73) \]

Using the fact that \( Mm_N < m_L (m_N - m_L < 0) \), we have

\[ \lim_{\nu_0 \to \infty} P_{\text{LOS}}(d_0) \frac{P_{\text{out,L}}(\gamma_T,\nu_0,\nu)}{P_{\text{out,N}}(\gamma_T,\nu_0,\nu)} = 0. \]
\[ (4) \]

Therefore

\[ P_{\text{out}}(\gamma_T,\nu_0,\nu) \approx \lim_{\nu_0 \to \infty} (1 - P_{\text{LOS}}(d_0)) \frac{P_{\text{out,N}}(\gamma_T,\nu_0,\nu)}{P_{\text{out,L}}(\gamma_T,\nu_0,\nu)}. \]
\[ (75) \]

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