On Partial Zero-Forcing Interference Cancellation in Uplink Cellular Networks

Wael Fatnassi and Zouheir Rezki
Electrical and Computer Engineering Department
University of Idaho, USA

Abstract

Partial zero forcing (PZF) represents an efficient and low-complexity technique for the uplink interference mitigation in cellular networks. In this paper, we address the outage probability of PZF interference cancellation technique at the base station (BS). We derive an exact closed-form expression of the outage probability, along with its a simplified asymptotic expression at high signal to noise ratio (SNR) regime. The high SNR analysis shows a diversity gain of \( M - N_c \), where \( M \) is the number of receive antennas at the BS and \( N_c \) is the number of cancelled interferers \( (N_c \leq M - 1) \). The high SNR analysis also demonstrates the tradeoff between interference cancellation and the remaining array gain. Numerical results show that PZF outperforms maximum ratio combining (MRC), i.e., a decoding scheme that does not cancel the interference and considered them as noise.

Keywords

Partial zero forcing (PZF), maximum ratio combining (MRC), interference mitigation, outage probability.

I. INTRODUCTION

Interference plays a crucial role in uplink cellular networks since it may drop the performance significantly, if it is not dealt with properly. More than any other single effect, interference can lead to quite catastrophic results at a typical receiver [1], [2]. In cellular networks, it is commonly accepted that the system is mostly interference-limited and the effect of noise is only marginal. That is, most yardstick of interest, including outage probability, capacity, delay, etc., are drastically impacted by interference. Network densification (also known today as dense
networks, or heterogenous networks) only makes the situation worst since in most cases only a little is known about the statistics of this interference, given that different users may utilize different codebooks and have different channel statistics. Clearly, in these realistic scenarios, information theoretical results and guidelines are particularly not directly applicable and may require careful adaptation. Therefore, interference mitigation in the context of dense networks is an appealing problematic that needs be seriously addressed.

Although interference cancellation has been applied in CDMA-based cellular networks e.g., [3], multiuser detection (MUD) has witnessed a phenomenal resurgence of interest by mid 90’s [4]. Among numerous MUD techniques, successive interference cancellation (SIC) is of particular interest due to its simplicity. For instance, in an uplink setting, the receiver decodes the strongest user (the user with the strongest channel) first, treating all other users’ signals as noise. Then, the receiver cancels out the contribution of the strongest user’s signal from the received signal. The receiver uses the newly obtained received signal to decode the second strongest user, cancels out its contribution from the last received signal. The receiver keeps doing so until all symbols are decoded. Obviously, this successive process has seen several variants, each trying to overcome the shortcomings of another, e.g., [1], [5]–[7]. However, SIC induces a high delay due to its sequential processing and is prone to error propagation, especially in absence of exact channel state information (CSI) at the receiver [8], [9]. To overcome these shortcomings, partial interference cancellation (PIC) has been proposed [10]. The key idea of PIC is that not the all the interference should be cancelled out and the process must be stopped if the reward resulting from cancelling an extra user is marginal. Hence, the receiver cancels part of interference, treating the remaining as noise. The number of cancelled interferers depends on their channel conditions. For instance, if the receiver has perfect CSI of a set of interferers and no CSI of other interferers, it may decode the first set via a SIC approach while treating the second set of interferers as noise [11]. In order to further enhance the performance and the accuracy of PIC, an optimal decision ordering can be potentially applied on the signal detection process which will correspondingly result to the decoding of the strongest user first, i.e. the user which experiences the best Signal-to-Interference-plus-Noise-Ratio (SINR) and/or Signal-to-Noise-Ratio (SNR). In general, users should be decoded in the order of their received powers [12].

In this paper, we study the impact of interference mitigation for uplink communications in
cellular networks, where the multi-antenna base station (BS) adopts partial zero forcing (PZF). Our channel model accounts for Rayleigh small-scale fading and shadowing to better capture outdoor cellular networks. We derive an exact closed-form expression of the outage probability, along with its a simplified asymptotic expression at high signal to noise ratio (SNR) regime. The high SNR analysis also demonstrates the tradeoff between interference cancellation and the remaining array gain. That is, one may cancel more interference by increasing $N_c$, but at the cost of reducing the array gain and vice versa. Our numerical results show that PZF outperforms maximum ratio combining (MRC), i.e., a decoding scheme that does not cancel the interference and considered it as noise.

II. SYSTEM MODEL

The network comprises $K+2$ nodes. It includes a base station $R_0$, a reference transmitter $T_0$, and $K$ interferers $T_i$, $1 \leq i \leq K$. Each transmitter has one single antenna while the base station has $M$ receive antennas. Our signal model is based on complex baseband model. We assume a flat fading channel as the wideband channel can be split into a set of parallel narrowband channels using orthogonalization techniques [12]. We assume that the receiver will cancel the interference from $N_c$ interfering transmitter ($N_c \leq K$) and the $K-N_c$ remaining will be considered as a noise. The complex channel coefficients of the $N_c$ interferers are assumed to be known at the receiver. The signal at each receive antenna is corrupted by an Additive White Gaussian Noise (AWGN) with zero mean and variance $\sigma^2$. We consider that separation between antennas is larger than half-wavelength. Consequently, there is no spatial correlation between receive antennas. During the transmission process and at an instant $t$, the $i^{th}$ node transmits the signal $s_i(t)$, with $E\left(s_i(t)^2\right) = P_i$, the transmitted power of user $i$. The channel vector between the $i^{th}$ transmitter and the base station is equal to $h_i = [h_{i1}, h_{i2}, ..., h_{iM}]^T$ where $h_{ji}$ is the complex channel coefficient between the $i^{th}$ transmitter and the $j^{th}$ receive antenna. Each channel gain $h_i$ accounts for Rayleigh fading and shadowing as described by

$$h_i = 10^{\eta_i/10} w_i, \quad 0 \leq i \leq K,$$

where $\eta_i$ is the shadowing factor and $w_i \sim \mathcal{CN}(0, I_M)$, is the fading vector. In the presence of log-normal shadowing, the $\{\eta_i\}$ are independent and identically distributed (i.i.d.) Gaussian with mean $\mu_s$ and variance $\sigma_s^2$. We denote by $x(t) = [x_1(t), x_2(t), ..., x_M(t)]^T$ the vector of signals
at \( M \) receive antennas,
\[
x(t) = \sum_{i=0}^{N_c} h_i s_i(t) + \left( \sum_{i=N_c+1}^{K} h_i s_i(t) + n(t) \right),
\]
where \( n(t) \) represents the AWGN vector, with \( n(t) \sim \mathbb{C} \mathbb{N}(0, \sigma^2 I_M) \), a circularly symmetric complex white Gaussian noise with 0 mean and covariance \( \sigma^2 I_M \).

### III. Partial Zero Forcing (PZF) Receiver

Now, the receiver may utilize a Zero-Forcing (ZF) based partial interference cancellation (PZF) decoding strategy to cancel the \( N_c \) interferers [13]. Accordingly, let us define \( H(N_c) = (h_1, \ldots, h_{N_c}) \in \mathbb{C}^{M \times N_c} \), which includes the effective channels from the \( N_c \) interfering nodes. Departing from (2), it can be easily verified that cancelling \( N_c \) interferers is equivalent to multiplying the received signal by \( (I - H(N_c) H(N_c)^\#) \). The next step is to decode \( s_0(t) \) in presence of \( K - N_c \) interferers and white Gaussian noise via a simple MRC. This can be done by multiplying the newly obtained signal, i.e., \( (I - H(N_c) H(N_c)^\#) x(t) \), by \( \frac{h_0^H (I - H(N_c) H(N_c)^\#) h_0}{\| (I - H(N_c) H(N_c)^\#) h_0 \|^2} \). Using properties of the pseudoinverse, it can be verified (c.f. appendix A) that PZF is equivalent to multiplying the received signal by a filter \( v(N_c)^H \) given by:
\[
v(N_c) = \frac{\left( I - H(N_c) H(N_c)^\# \right) h_0}{\| \left( I - H(N_c) H(N_c)^\# \right) h_0 \|^2},
\]
where \( A^\# \) designates the pseudoinverse of \( A \). Observe that \( v(N_c) \) in (3) reduces to maximum ratio combining (MRC) when \( N_c = 0 \) and to full zero forcing when \( N_c = M - 1 \).

### IV. Outage Performance

As the receiver does not know the actual CSI of the \( K - N_c \) interferers and only knows their statistics, reliable communications are subject to an outage which can be characterized by deriving first the resultant signal-to-interference plus noise ratio (SINR) \( \gamma \). This is computed in the following

**Lemma 1.** For the PZF interference cancellation described above, the resulting SINR after cancelling \( N_c \) interferers at the base station is given by
\[
\gamma(\rho_0, \rho, N_c) = \frac{\rho_0 |v(N_c)^H h_0|^2}{\sum_{i=N_c+1}^{K} \rho_i |v(N_c)^H h_i|^2 + 1},
\]
where $\rho_i = \frac{P_i}{\sigma^2}$ is the SNR of the $i^{th}$ transmitter and $\rho = [\rho_{N_c+1}, \ldots, \rho_K]$.

**Proof:** For convenience, the proof is presented in Appendix B.

An upper bound on the outage probability is defined as the probability that the received SINR is below a given threshold $\gamma_T$ for a given $\rho_0$, $\rho$ and $N_c$, i.e.,

$$P_{out}(\gamma_T, \rho_0, \rho, N_c) \triangleq P_T\left(\gamma(\rho_0, \rho, N_c) \leq \gamma_T\right).$$  \hfill (5)

Next, we derive an explicit closed-form of $P_{out}(\gamma_T, \rho_0, \rho, N_c)$ as given in theorem 1.

**Theorem 1.** For the PZF interference cancellation described above, an upper bound on the outage probability is given by

$$P_{out}(\gamma_T, \rho_0, \rho, N_c) = \frac{2^M}{2^M - N_c} \sum_{k=0}^{\infty} \frac{(-1)^k (2\gamma_T \rho_0)^k}{k!(M - N_c + k)} f_{\mu,s} (M - N_c + k) \times \sum_{t=0}^{M - N_c + k} \binom{M - N_c + k}{t} t! \prod_{j=t+1}^{K} \frac{(\rho_j)^{t_j}}{t_j!} \Gamma(t_j + 1) f_{\mu,s} (t_j)$$

where $f_{\mu,s} (t_j) = e^{-\frac{(t_j \sigma_s)^2}{2}} - e^{-\mu_j t_j}$.

**Proof:** For convenience, the proof is presented in Appendix C.

In the absence of shadowing and when $K = N_c$, (6) reduces to

$$P_{out}(\gamma_T, \rho_0, N_c) = \gamma(M - N_c, \gamma_T \rho_0) \frac{(M - N_c - 1)!}{(M - N_c - 1)!},$$  \hfill (7)

where $\gamma(a,x) = \int_0^x t^{a-1} e^{-t} \, dt$ is the lower incomplete Gamma function.

Next, we study the high SNR regime of the transmitter of interest, and derive an asymptotic expression for the outage probability, which enables the characterization of the achievable diversity order. Specifically, we characterize the two key performance parameters dictating the outage probability in the high SNR regime, i.e., the diversity gain $G_d$ and the array gain $G_a(\gamma_T, \rho, N_c)$

$$P_{out}^\infty(\gamma_T, \rho_0, \rho, N_c) = G_a(\gamma_T, \rho, N_c) \rho_0^{-G_d} + o(\rho_0^{-G_d})$$

where $o(\rho_0^{-G_d})$ is a function of $\rho_0$ such that $\lim_{\rho_0 \to \infty} \frac{o(\rho_0^{-G_d})}{\rho_0^{-G_d}} = 0$. 

Corollary 1. In the high SNR regime, i.e., $\rho_0 \rightarrow \infty$, the outage probability can be expressed as

$$P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c) = G_a(\gamma_T, \rho, N_c) \rho_0^{-G_d} + o(\rho_0^{-G_d}),$$

(9)

where

$$G_d = M - N_c,$$

(10)

$$G_a(\gamma_T, \rho, N_c) = \frac{\gamma_T^{M-N_c}}{(M-N_c)!} f_{\mu_s, \sigma_s}(M-N_c) \sum_{t=0}^{M-N_c} \binom{M-N_c}{t} t!$$

$$\times \sum_{\sum_{j=1}^{K-N_c} t_j = t} \prod_{i=N_c+1}^{K} \frac{t_j^{t_j}}{t_j!} \Gamma(t_j + 1) f_{\mu_s, \sigma_s}(t_j).$$

(11)

Proof: The proof is presented in Appendix D.

Corollary 1 presents the asymptotic expression for the outage probability. It points out the tradeoff between interference cancellation from one hand and the diversity and the array gains from another hand. By increasing $N_c$, we decrease the interference, but also decrease the array gain and the diversity order.
Figure 2. Array gain in dB vs $N_c$, using the analytic expression (11). The network comprises 12 users and a receiver with 13 antennas. The transmit SNRs of the interferers are equals to 20 dB ($\rho_i = 20$ dB, $1 \leq i \leq K$). The threshold ($\gamma_T$) is equal to 10 dB, $\mu_s = 0$ and $\sigma_s = 7$ dB.

V. NUMERICAL RESULTS

Figure 1 shows the outage probability as function of the transmit SNR $\rho_0$ of the reference transmitter. This figure illustrates that Monte Carlo simulations match the analytical expression in (6). The asymptotic expression given in (9) provides accurate prediction of the outage probability in the high SNR regime. It can be seen from this figure that there exists a tradeoff between interference cancellation and the diversity order. By increasing $N_c$, we decrease the interference, but also decrease the diversity order. Figure 2 shows the array gain as function of $N_c$. We can see that by increasing $N_c$, we decrease the array gain.

While Figs. 1 and 2 highlight the drawback of PZF at high SNR, Fig. 3 shows its benefit when the outage probability is plotted versus the SINR threshold $\gamma_T$. As illustrated in Fig. 3, PIC provides a consistent gain over MRC. For instance at outage probability equal to $10^{-1}$, cancelling 10 interferers provides a 10.5 dB gain over MRC. Although the PZF’s performance with $N_c = 10$ is below the single-user outage performance, it is clear that increasing $N_c$ will close the gap. The ZF receiver outperforms both MRC and PZF. However, ZF requires perfect CSI of all users’channels which might not be feasible.
Figure 3. Outage probability versus the threshold $\gamma_T$ in dB, with partial zero-forcing (PZF), for a network with 19 users and a receiver with 20 antennas. The transmit SNRs are equals to 20 dB ($\rho_i = 20$ dB, $0 \leq i \leq K$). To account for shadowing, we set $\mu_s = 0$ and $\sigma_s = 4$ dB. The lowest curve corresponds to the idealistic case of perfectly canceled interference (ZF), i.e., the single-user outage lower bound. The upper curve corresponds to MRC, where no interference mitigation technique is used at all. The middle curve corresponds to PZF where only 10 users are cancelled out whereas the remaining are treated as noise.

VI. CONCLUSION

In this paper, we studied the impact of interference mitigation in uplink cellular networks. We used PZF receive filter. We derived the probability of outage and highlighted the corresponding diversity and array gains. We showed that PZF outperforms MRC, i.e., a decoding scheme that does not cancel the interference and considered them as noise. By increasing the number of cancelled interferers, the diversity order and the array gain decrease. This happens because PZF uses some of the receiver antennas to cancel the interference and exploits the remaining antennas to provide diversity and array gains.

APPENDIX A

The vector of received signals at $M$ receive antennas is equal to:

$$\mathbf{x}(t) = \sum_{i=0}^{N_c} \mathbf{h}_i s_i(t) + \left( \sum_{i=N_c+1}^{K} \mathbf{h}_i s_i(t) + \mathbf{n}(t) \right)$$

$$= \mathbf{h}_0 s_0(t) + \mathbf{H}(N_c) \mathbf{S}_1(t) + \mathbf{G}(K-N_c) \mathbf{S}_2(t) + \mathbf{n}(t),$$  \hspace{1cm} \text{(12)}

where $\mathbf{S}_1(t) = (s_1(t), \ldots, s_{N_c}(t))$, $\mathbf{S}_2(t) = (s_{N_c+1}(t), \ldots, s_K(t))$, $\mathbf{H}(N_c) = (h_1(t), \ldots, h_{N_c}(t))$, and $\mathbf{G}(K-N_c) = (h_{N_c+1}(t), \ldots, h_K(t))$. $\mathbf{S}_1(t)$ and $\mathbf{H}(N_c)$ include the transmitted signals
and the effective channels from the \( N_c \) interfering nodes, while \( S_2 (t) \) and \( G (K - N_c) \) include the transmitted signals and the effective channels from the remaining interferers. To cancel the \( N_c \) interferers, we multiply \( x (t) \) by a matrix \( L^H (N_c) \in \mathbb{C}^{N_c \times M} \). After the multiplication, we obtain the following expression

\[
\tilde{x} (t) = L^H (N_c) x (t) = L^H (N_c) h_0 s_0 (t) + L^H (N_c) H (N_c) S_1 (t) + L^H (N_c) \left( G (K - N_c) S_2 (t) + n (t) \right). \tag{13}
\]

Departing from (13), the receiver first decodes \( N_c \) interferers by choosing \( L^H (N_c) \) such that

\[
L^H (N_c) H (N_c) = I_{N_c}, \tag{14}
\]

where \( I_{N_c} \in \mathbb{C}^{(N_c)} \) is the identity. Considering (14), \( L^H (N_c) \) is the pseudoinverse of \( H (N_c) \). Since \( N_c < M \), the matrix \( H (N_c) H (N_c) \) is invertible. Hence, the pseudoinverse of \( H (N_c) \) can be computed as

\[
L^H (N_c) = \left( H (N_c) H (N_c) \right)^{-1} H (N_c)^H. \tag{15}
\]

Next, the receiver cancels the \( N_c \) interferers as follows

\[
x (t) - H (N_c) \tilde{x} (t) = \left( I_{N_c} - H (N_c) L^H (N_c) \right) h_0 s_0 (t) + \left( I_{N_c} - H (N_c) L^H (N_c) \right) \left( G (K - N_c) S_2 (t) + n (t) \right). \tag{16}
\]

The next step is to decode \( s_0 (t) \) in presence of \((K - N_c)\) interferers and white noise via a simple MRC. This can be done by multiplying the newly obtained signal, i.e., \( \left( I_{N_c} - H (N_c) L^H (N_c) \right) x (t) \) by \( L^H_{mrc} = \frac{h_0^H (I_{N_c} - H (N_c) L^H (N_c))}{\| (I_{N_c} - H (N_c) L^H (N_c)) h_0 \|} \), where \( L_{mrc} \) is the weight vector for MRC. Using properties of the pseudoinverse, it can be easily verified that

\[
(I_{N_c} - H (N_c) L^H (N_c))^H (I_{N_c} - H (N_c) L^H (N_c)) = \left( I_{N_c} - H (N_c) L^H (N_c) \right)^H, \tag{17}
\]

Using (17), we obtain

\[
L^H_{mrc} \left( I_{N_c} - H (N_c) L^H (N_c) \right) x (t) = v (N_c)^H x (t), \tag{18}
\]

where \( v (N_c) = \frac{(I_{N_c} - H (N_c) L^H (N_c)) h_0}{\| (I_{N_c} - H (N_c) L^H (N_c)) h_0 \|} \).
APPENDIX B
PROOF OF LEMMA 1

Cancelling $N_c$ interferers and decoding $s_0(t)$ in presence of $(K-N_c)$ and white Gaussian noise via a simple MRC is equivalent to multiplying the received signal $x(t)$ by $v^H(N_c)$ as follows

$$v^H(N_c)x(t) = v^H(N_c)h_0s_0(t) + v^H(N_c)G(K-N_c)S_2(t) + v^H(N_c)n(t).$$ \hspace{1cm} (19)

The power of signal of interest ($P_S$) is equal to

$$P_S = E_{s_0(t)}(v^H(N_c)h_0s_0(t)s_0(t)h_0^Hv(N_c))$$

$$= P_0 |v^H(N_c)h_0|^2,$$ \hspace{1cm} (20)

where $E_X(.)$ denotes the expectation with respect to $X$. The power of $K-N_c$ interferers $P_I$ is equal to

$$P_I = E_{s_i(t)}(v^H(N_c)G(K-N_c)S_2(t)S_2^H(t)G^H(K-N_c)v(N_c)), \quad i \in (N_c+1, \ldots, K)$$

$$= v^H(N_c)G(K-N_c)D G^H(K-N_c)v(N_c),$$ \hspace{1cm} (21)

where $D = E_{s_i(t)}(S_2(t)S_2^H(t)) = \text{diag}(P_{N_c+1}, \ldots, P_K) \in \mathbb{C}^{(K-N_c) \times (K-N_c)}$ is a diagonal matrix containing the powers of $K-N_c$ interferers. Hence, (21) becomes

$$P_I = \sum_{i=N_c+1}^{K} P_i |v^H(N_c)h_i|^2. \hspace{1cm} (22)$$

On the other hand, the power of noise $P_N$ is equal to

$$P_N = E_{n(t)}(v^H(N_c)n(t)n^H(t)v(N_c))$$

$$= \sigma^2 \|v(N_c)\|^2 = \sigma^2.$$ \hspace{1cm} (23)

Hence, the SINR is expressed as follows

$$\gamma(\rho_0, \rho, N_c) = \frac{P_S}{P_I + P_N} = \frac{P_0 |v(N_c)^H h_0|^2}{\sum_{i=N_c+1}^{K} P_i |v(N_c)^H h_i|^2 + \sigma^2}$$

$$= \frac{\rho_0 |v(N_c)^H h_0|^2}{\sum_{i=N_c+1}^{K} \rho_i |v(N_c)^H h_i|^2 + 1},$$ \hspace{1cm} (24)
where \( \rho_i = \frac{P_i}{\sigma^2} \) and \( \rho = [\rho_{N_c+1}, \ldots, \rho_K] \).

**APPENDIX C**

**PROOF OF THEOREM 1**

We can express the SINR in (24) differently as follows

\[
\gamma (\rho_0, \rho, N_c) = \frac{\rho_0 \beta_0 |v(N_c)Hw_0|^2}{\sum_{i=N_c+1}^{K} \rho_i \beta_i |v(N_c)Hw_i|^2 + 1},
\]

(25)

where \( \beta_i = 10^{n_i} \), and \( v(N_c) = \frac{(I_{N_c}-H(N_c)H(N_c)^\prime)h_0}{\| (I_{N_c}-H(N_c)H(N_c)^\prime)h_0 \|} = \frac{(I_{N_c}-W(N_c)W(N_c)^\prime)w_0}{\| (I_{N_c}-W(N_c)W(N_c)^\prime)w_0 \|} \), with \( W = (w_1, \ldots, w_{N_c}) \). First, we derive the upper bound on the outage probability conditioned on shadowing vector \( \beta = [\beta_0 = 10^{n_0}, \beta_{N_c+1} = 10^{n_{N_c+1}}, \ldots, \beta_K = 10^{n_K}] \), which can be written as follows

\[
P_{out} (\gamma_T, \rho_0, \rho, N_c|\beta) = Pr (\gamma (\rho_0, \rho, N_c) \leq \gamma_T|\beta). \]

(26)

We define 2 RVs \( R \) and \( Z \) which are expressed as follows

\[
R = \gamma_T^{-1} \rho_0 \beta_0 S_0 (N_c),
\]

(27)

\[
Z = [Z_{N_c+1}, \ldots, Z_K],
\]

(28)

where \( Z_i = \rho_i \beta_i S_i (N_c), \ N_c+1 \leq i \leq K \), and \( S_i (N_c) = |v(N_c)Hw_i|^2 \). Then, the conditioned outage probability may be expressed as

\[
P_{out} (\gamma_T, \rho_0, \rho, N_c|\beta) = 1 - Pr \left( R \geq \sum_{i=N_c+1}^{K} Z_i + 1|\beta \right)
\]

\[
= 1 - P (\gamma_T, \rho_0, \rho, N_c|\beta).
\]

(29)

Conditioned on \( \beta \), let \( f_R (r) \) denotes the probability density function (pdf) of \( R \) and \( f_Z (z) \) denotes the joint pdf of \( Z \). Using these definitions, \( P_{out} (\gamma_T, \rho_0, \rho, N_c|\beta) \) is expressed as follows

\[
P (\gamma_T, \rho_0, \rho, N_c|\beta) = \int \cdots \int \int_{R_{N_c}^{K-N_c}} f_Z (z) \left( \int_{\sum_{i=N_c+1}^{K} z_i + 1}^{\infty} f_R (r) dr \right) dz.
\]

(30)

The RVs \( S_0 (N_c) \) and \( S_i (N_c) (i = N_c + 1, \ldots, K) \) are distributed as chi-squared RVs \([14]\) of \( 2(M - N_c) \) and 2 degrees of freedom, respectively, i.e., \( S_0 (N_c) \sim \chi^2_{2(M-N_c)} \) and \( S_i (N_c) \sim \chi^2_2 \).
\( i = N_c + 1, \ldots, K \). Thus, the inner integral in (30) is equal to

\[
\int_{(\sum_{i=N_c+1}^{K} z_i + 1)}^\infty f_R(r) \, dr = \frac{\Gamma\left(M - N_c, \frac{\sum_{i=N_c+1}^{K} z_i + 1}{\rho_0 \beta_0}\right)}{(M - N_c - 1)!},
\]

(31)

where \( \Gamma(a, x) = \int_{x}^{\infty} t^{a-1} e^{-t} \, dt \) is the upper incomplete Gamma function. Using [15, eq. (8.354.2)], we can express (31) differently as follows

\[
\int_{(\sum_{i=N_c+1}^{K} z_i + 1)}^\infty f_R(r) \, dr = 1 - \frac{1}{(M - N_c - 1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{M - N_c + k}{\rho_0 \beta_0}\right)^{M - N_c + k}}{k! (M - N_c + k)} \left(1 + \sum_{i=N_c+1}^{K} z_i\right)^{M - N_c + k}.
\]

(32)

Since \( (M - N_c + k) \) is integer, we can use the Binomial theorem to develop the inner expression in (32) as follows

\[
\left(1 + \sum_{i=N_c+1}^{K} z_i\right)^{M - N_c + k} = \sum_{t=0}^{M - N_c + k} \binom{M - N_c + k}{t} \left(\sum_{i=N_c+1}^{K} z_i\right)^{t}.
\]

(33)

We use the Multinomial expansion to develop the inner expression in (33)

\[
\left(\sum_{i=N_c+1}^{K} z_i\right)^{t} = t! \sum_{\sum_{j=N_c+1}^{K} t_j = t} \left(\prod_{i=N_c+1}^{K} \frac{z_i^{t_j}}{t_j!}\right) = t! \sum_{\sum_{j=1}^{K} t_j = t} \left(\prod_{j=1}^{K} \frac{z_i^{t_j}}{t_j!}\right).
\]

(34)

Given \( \beta, \{Z_i\}_{i=N_c+1}^{K} \) are independent. So, \( f_Z(z) \) can be written as \( \prod_{i=N_c+1}^{K} f_{Z_i}(z_i) \). Considering this and substituting (34) and (33) into (32), (30) becomes

\[
P(\gamma_T, \rho_0, \rho, N_c|\beta) = 1 - \frac{1}{(M - N_c - 1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{M - N_c + k}{\rho_0 \beta_0}\right)^{M - N_c + k}}{k! (M - N_c + k)} \sum_{t=0}^{M - N_c + k} \binom{M - N_c + k}{t} t!
\]

\[
\times \sum_{\sum_{j=1}^{K} t_j = t}^{K-N_c} \prod_{j=1}^{K} \int_{0}^{\infty} \frac{z_i^{t_j}}{t_j!} f_{Z_i}(z_i) \, dz_i.
\]

(35)

We have \( f_{Z_i}(z_i) = \frac{1}{\rho_i \beta_i} e^{-\frac{z_i}{\rho_i \beta_i}} \). Using this expression, the inner integral in (35) becomes

\[
F_{(t_j, \rho_0)}(\beta_i) = \int_{0}^{\infty} \frac{z_i^{t_j}}{t_j!} f_{Z_i}(z_i) \, dz_i = \frac{(\rho_i \beta_i)^{t_j}}{t_j!} \Gamma(t_j + 1),
\]

(36)
Substituting (36) into (35), the conditioned outage probability becomes

\[
P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c | \beta) = \frac{1}{(M - N_c - 1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\gamma_T}{\rho_0 \beta_0}\right)^{M - N_c + k}}{k! (M - N_c + k)} \sum_{t=0}^{M - N_c + k} \left(\frac{M - N_c + k}{t}\right) t!
\times \sum_{K - N_c \leq t_j = t}^{K} \prod_{i=N_c + 1}^{K} F_{(t_j, \rho_i)}(\beta_i).
\] (37)

Next, we derive the conditioned outage probability \(P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c | \beta_0)\), by taking an expectation with respect to \(\beta_s = [\beta_{N_c+1}, \ldots, \beta_K]\). Since \(\{\beta_i\}_{i=N_c+1}^{K}\) are independent

\[
E_{\beta_s} \left( \prod_{i=N_c+1}^{K} F_{(t_j, \rho_i)}(\beta_i) \right) = \prod_{i=N_c+1}^{K} \left( E_{\beta_i} \left( F_{(t_j, \rho_i)}(\beta_i) \right) \right)
= \prod_{i=N_c+1}^{K} \left( \int_0^\infty f_{\beta_i}(\beta_i) F_{(t_j, \rho_i)}(\beta_i) \, d\beta_i \right),
\] (38)

where

\[
f_{\beta_i}(\beta_i) = \frac{1}{\beta_i \sigma_s \sqrt{2\pi}} e^{-\frac{(\ln \beta_i - \mu_s)^2}{2\sigma_s^2}}.
\] (39)

Using [15, eq. (3.323.2)], (39) and by making a change of variable \(r_i = \ln \beta_i\) to calculate the inner integral in (38), we obtain the expression of \(P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c | \beta_0)\). In the final step, we derive \(P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c)\) by taking the expectation with respect to \(\beta_0\) following a similar technique.

**APPENDIX D**

**PROOF OF COROLLARY 1**

Using the results from theorem 1, the outage probability is expressed as follows

\[
P_{\text{out}}(\gamma_T, \rho_0, \rho, N_c) = \frac{1}{(M - N_c - 1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\gamma_T}{\rho_0}\right)^{M - N_c + k}}{k! (M - N_c + k)} \frac{f_{\mu_s, \sigma_s}(M - N_c + k)}{(M - N_c + k)!} \sum_{t=0}^{M - N_c + k} \left(\frac{M - N_c + k}{t}\right) t!
\times \sum_{\sum_{j=1}^{K - N_c} t_j = t}^{K \geq N_c} \prod_{i=N_c + 1}^{K} \left( \frac{\rho_i}{t_j} \right)^{t_j} \Gamma(t_j + 1) f_{\mu_s, \sigma_s}(t_j).
\] (40)

In the high SNR regime, \(\rho_0 \to \infty\) and thus \(\frac{\gamma_T}{\rho_0} \to 0\). Therefore, the asymptotic expression for the outage probability \(P_{\text{out}}^\infty(\gamma_T, \rho_0, \rho, N_c)\) is obtained by setting \(k = 0\), i.e., the first term of the sum.
REFERENCES


