Reliability enhancement of Smart Metering System Using Millimeter Wave Technology

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Abstract

Millimeter wave (mmWave) technology has been advocated as a promising infrastructure to provide reliable and high data-rate communications, both in indoor and outdoor environments. In this paper, we extend the application of mmWave to the uplink communication between smart meters (SMs) and a gateway. Such a communication is subject to interference from SMs belonging to adjacent networks and blockage caused by human bodies, for instance. Using a three-dimension (3D) stochastic blockage model, we derive the outage probability. When human-body blockage is neglected, the high-signal to noise ratio (SNR) analysis shows a diversity gain of \((m_L M)\), where \(m_L\) is the Nakagami-fading parameter of the line of sight (LOS) of the reference transmitter’s channel and \(M\) is the number of receive antennas at the gateway. Accounting for human-body blockage, the diversity gain reduces to \((m_N M)\) where \(m_N\) is the Nakagami-fading parameter of the non line of sight (NLOS) of the reference transmitter’s channel. Our analysis shows that the probability that an SM is in LOS decays exponentially with the link length and the density of blockages. Although at high SNR blockage reduces the diversity gain, our numerical results show that blockage may decrease the outage probability at finite SNR.

Keywords

MmWave propagation channel, indoor radio channel, Nakagami fading, shadowing, Poisson point process.

I. INTRODUCTION

A. Motivation and Background

SG is considered as an intelligent network for delivering electrical power systems [1]. It combines the transmission of electricity and information in order to improve the quality of electricity transmission [2]. It includes a variety of operational and energy measures including
smart meters (SMs). Generally, SMS consists of metering and communication infrastructures [3]. The communication infrastructures of the SG may include wireline technologies such as power line communication (PLC) or wireless communications. However, the PLC faces many technical challenges like the unexpected propagation characteristics of transmission and distribution lines, strong electromagnetic interference and higher signal losses [3]. Hence why wireless communication has been incorporated in the SG as it brings several advantages in terms of installation, coverage and high flexibility.

Standards such as WiMax, UMTS, LTE and LTE-A have been already used for SG communications, in an outdoor environment. Likewise, standards such as IEEE 802.11, IEEE 802.15 based Wi-Fi, and Wireless Personal Area Network (WPAN) have been used for an indoor environment. Also, several industrial standards are based on IEEE 802.15.4 in order to perform monitoring and control applications. One of the most widely adopted standard in this class is ZigBee due to its extended network capabilities [4].

However, despite its numerous advantages, wireless communications in the SG suffer many challenges such as limited bandwidth and sensitivity to interference [5], to cite only a few. In that sense, the transmission at mmWave frequencies seems to be a promising alternative for SG due to its immunity against interference and its free wideband spectrum [6]. In addition, the management of SG power resources have the following requirements: Real-time processing of large volume of data and lower transmission latencies with reliable communication over the network [7]. Evidently, mmWave technology meets these requirements as one of the available technology options for reliable, secure and cost effective operation of SG [7]. Furthermore, SMS applications are short range in nature which is suitable to mmWave technology. There are two commercial standards such as Wireless HD [8] and IEEE 802.11ad [9] which are dedicated to applications using mmWave technology. Since it targets indoor environments, the last standard could be a candidate for smart metering system (SMS).

B. A brief Literature Survey

Recent work have investigated mmWave networks using stochastic geometry tools to analyze the coverage probability [10]. In [11], a closed-form expression was derived for the outage probability conditioned on the network geometry, in indoor wearable settings. Also, [11] has analysed a finite network with finite number of interferers. In [12], a similar approach as in
[11] is adopted to obtain the outage probability conditioned on the network geometry in Ad Hoc networks. An extension of such an approach to frequency-hopping network is considered in [13]. Although there is considerable progress in channel modeling for mmWave [14]–[17], the investigation on blockage modeling has been limited. In [15], a Boolean scheme has been used to model the blockage due to buildings in urban cellular network. However, the main limitation of this work is that only the direct propagation is considered and the reflected signals are ignored. Moreover, it was assumed that each link experiences an independent and identically distributed (i.i.d.) small-scale Rayleigh fading which does not seem to be a good statistical model for mmWave. Another ball-based blocking model was proposed in [16] to capture, once, again blockages due to building in outdoor environment. Using this model, the coverage probability and the capacity were derived and were compared to real building data obtained from Google Map [18]. In [19], the authors modeled human bodies as 3D cylinders with fixed heights and diameters. This blockage model was incorporated in stochastic geometry framework to derive the coverage probability. However, considering fixed heights and diameters does not seem to be reasonable in finite-size geometry.

We note that mmWave communications are expected to operate in shorter distances and in crowded environments [17]. In this case, human bodies can act as blockers to transmitted signals. Clearly, an accurate blockage model must account for the heights of transceivers, their separation, the dimension of blocking objects’s distributions. None of prior works have addressed these issues in their models.

C. Contribution

In this paper, we consider a mmWave SMS consisting of a set of SMs communicating with a gateway. We use the term gateway instead of the receiver as it will send the SMs’data to the service provider via UMTS, 4G, or WiMax. All nodes are located on the same floor of a building, for instance. These nodes may include water, heat and electricity meters. However, in each floor, we may have one gateway or more to receive the data from the nodes belonging to the same floor. Such a communication is subject to interference among the SMs belonging to the same gateway and other SMs belonging to different gateways. Motivated by the work in [20], our channel model includes path loss and shadowing. It also includes a Nakagami fast fading component which has been justified by [21]. A specific feature of SMS is that the transmitter sensors are kept at their
basic functionalities. That is, they are in transmit-only mode, with no channel estimation, no power control and no awareness of the environment. Obviously, although this feature keeps the most complexity at the receiver side, it increases the probability of collision, thus compromising SMS reliability. To overcome this limitation, our model comprises an Aloha-like medium access protocol, where each SM transmits with a certain probability. At the gateway, a maximum ratio combining (MRC) is used. For this broad setting, our contributions are as follows:

- We derive the outage probability along with the diversity gain of the system, assuming first that the effect of human-body blockage is negligible.
- We propose a general and tractable model that incorporates human-body blockages. In this model, humans are represented as cylinders whose centers follow the Poisson point process (PPP) with arbitrary heights and diameters.
- Using tools from stochastic geometry, we derived the probability of line of sight (LOS) between each SM and the gateway as function of the separation between the SM and the gateway and their heights.
- We derived the probability of outage accounting for human-body blockage.
D. Outline

The remainder of the paper is organized as follows. Section II discusses the system model. Section III presents the outage probability without blockages and Section IV presents the outage probability with blockages. Numerical results and their interpretations are presented in Section V. Section VI concludes the paper.

II. System Model

The network comprises $K + 2$ nodes. It includes a gateway $R_0$, a reference smart meter $T_0$ and $K$ interferers $T_i$, $1 \leq i \leq K$. Each node has a fixed location. Each transmitter has a single antenna while the gateway has $M$ receive antennas as shown in Fig. 1. Our signal model is based on complex baseband model. Although the mmWave spectrum is generally broadband, we assume a flat fading channel since the wideband channel can be split into a set of parallel narrowband channels using orthogonalization techniques [22]. The complex channel coefficients are assumed to be known at the receiver. The signal at each receive antenna is corrupted by an Additive White Gaussian Noise (AWGN) with zero mean and variance $\sigma^2$. We consider that separation between antennas are larger than half-wavelength. Consequently, there is no spatial correlation between receive antennas. During the transmission process and at an instant $t$, the $i^{th}$ node transmits the signal $s_i(t)$ (we assume $E(s_i(t)^2) = P_i$, the transmitted power of SM $i$). The channel vector between the $i^{th}$ transmitter and the gateway is equal to $h_i = [h_{1i}, h_{2i}, ..., h_{Mi}]^T$ where $h_{ji}$ is the complex channel coefficient between the $i^{th}$ transmitter and the $j^{th}$ receive antenna. Each channel gain $h_i$ accounts for Nakagami fast fading, path loss and shadowing as described by

$$h_i = \begin{cases} 
\beta_{0,L}^{-1/2} 10^{\eta_i/10} w_0, & i = 0 \\
I_i \beta_{i,L}^{-1/2} 10^{\eta_i/10} w_i, & 1 \leq i \leq K,
\end{cases} \tag{1}
$$

where $\beta_{i,L} = \beta_{i,L}(d_i) = \left(\frac{d_i}{d_{ref}}\right)^{-\alpha_L}$. $1$ is the path loss when the transmitter is in line of sight (LOS) which is expressed as a function of $d_i$ (the distance between the $i^{th}$ transmitter and the receiver), $d_{ref}$ is the reference distance and $\alpha_L$ is the attenuation power-law exponent LOS case, $\eta_i$ is the shadowing factor and $w_i = [w_{1i}, w_{2i}, ..., w_{Mi}]^T$ is the fading vector consisting

\footnote{Although $\beta_{i,L}(d_i)$ depends on $d_i$, we will write it simply as $\beta_{i,L}$ for convenience.}
of independent and identically distributed (i.i.d.) \(^2\) Nakagami-m RVs with fading parameter and spread parameter in LOS \(m_L\) and \(\Omega_L\), respectively. In the presence of log-normal shadowing, the \(\{\eta_i\}\) are i.i.d. Gaussian with mean \(\mu_s\) and variance \(\sigma^2_s\). We assume without loss of generality that \(m_L\) is constant over the antenna array. We note that the channel model in (1) has been widely used in the mmWave literature, e.g., [23]. We assume that path loss and shadowing between the transmitter and each receive antenna are the same. For the multiple access strategy, we suppose an Aloha medium access control (MAC) protocol. The \(i^{th}\) interferer transmits with a probability \(p_i\). We denote by \(I_i\) a Bernoulli RV which has the following probability:

\[
P(I_i) = \begin{cases} p_i, & \text{if } I_i = 1 \\ 1 - p_i, & \text{if } I_i = 0. \end{cases} \tag{2}
\]

We denote by \(\mathbf{x}(t) = [x_1(t), x_2(t), ..., x_M(t)]^T\) the vector of signals at \(M\) receive antennas.

\[
\mathbf{x}(t) = \sum_{i=0}^{K} h_is_i(t) + \mathbf{n}(t), \tag{3}
\]

where \(\mathbf{n}(t)\) represents the AWGN vector, with \(\mathbf{n}(t) \sim \mathcal{CN}(0, \sigma^2 I_M)\), a circularly symmetric white Gaussian noise with 0 mean and covariance \(\sigma^2 I_M\).

We assume a MRC-based receiver. Therefore, the output signal \(\mathbf{y}(t)\) of an M-element antenna array operating in the presence of \(K\) interferers is equal to:

\[
\mathbf{y}(t) = \mathbf{L}_m\mathbf{r}(t), \tag{4}
\]

where \(\mathbf{L}_m\) is the weight vector for MRC. Taking \(\mathbf{L}_m = \mathbf{h}_0\) leads to the following equation:

\[
\mathbf{y}(t) = \mathbf{h}_0^H \mathbf{h}_0 s_0(t) + \sum_{i=1}^{K} \mathbf{h}_i^H \mathbf{h}_i s_i(t) + \mathbf{h}_0^H \mathbf{n}(t). \tag{5}
\]

III. OUTAGE PERFORMANCE

From (5), it is clear that decoding interference as noise makes the signal to interference plus noise ratio (SINR) as the yardstick of interest for analysing the outage probability of SMS. The

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\(^2\)Note that our derivation still holds when these components are independent and non-identically distributed. However, the analytical results will be more complicated
SINR can thus be computed as
\[
\gamma (\nu_0, \nu) = \frac{P_0(h_0^H h_0)^2}{\sum_{i=1}^{K} P_i(h_0^H h_i^H h_0) + \sigma^2(h_0^H h_0)}
\]
\[
\gamma = \frac{P_0 \zeta_0}{\sum_{i=1}^{K} P_i I_i^2 \zeta_i + \sigma^2} = \frac{\nu_0 \zeta_0}{\sum_{i=1}^{K} I_i^2 \nu_i \zeta_i + 1},
\]
where \( \zeta_i = \beta_i L \lambda_1^{m_1} b_i, \ b_i = \frac{w_0^H w_i w_i^H w_0}{w_0^H w_0}, \ \nu_0 = \frac{P_0}{\sigma^2}, \) and \( \nu = [\nu_1 = \frac{P_1}{\sigma^2}, \ldots, \nu_K = \frac{P_K}{\sigma^2}]. \) In (6), \( \zeta_i \) represents the instantaneous channel between the \( i \)-th SM and the gateway, \( \nu_0 \) represent the transmit SNR of the SM of interest and \( \nu_i, \ 1 \leq i \leq K, \) represents the transmit SNR at each interfering SM. The outage probability is defined as the probability that the received SINR is below a given threshold \( \gamma_T \) for a given \( \nu_0 \) and \( \nu, \) i.e.,
\[
P_{\text{out}} (\gamma_T, \nu_0, \nu) \triangleq \Pr \left( \gamma (\nu_0, \nu) \leq \gamma_T \right).
\]
In this subsection, we derive the conditioned outage probability \( P_{\text{out}} (\gamma_T, \nu_0, \nu|\chi) \) for a given shadowing factor vector \( \chi = [\chi_0 = 10^{\eta_0}, \ldots, \chi_K = 10^{\eta_K}] \). We have the following key result:

**Theorem 1.** The conditioned outage probability of SMS can be expressed as follows:
\[
P_{\text{out}} (\gamma_T, \nu_0, \nu|\chi) = \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} T_0^m L^{M+j} \Gamma (mL M + j) \left[ \prod_{i=1}^{K} (1 - p_i) + \sum_{S \subseteq E} \left( \prod_{i \in S} p_i (1 - p_i) \right) \right]
\]
\[
\times \prod_{i \in S} \left( \psi_{\text{min}}^i \right)^m L^{N_S} \sum_{k=0}^\infty \frac{\delta_k}{\Gamma (N_S m_L + k) \Gamma (-m_L M - j)} G_{2,1}^{1,2} \left( -N_S m_L - k + 1, m_L M + j + 1 \mid \psi_{\text{min}} \right)
\]
where the second sum in (8) is overall non empty subsets \( E, \ N_S = \text{card}(S) \) is the number of elements in subset \( S, \ \psi_i = \frac{\nu_i \lambda_i^{m_i} \chi_i}{m_i}, \ \psi_{\text{min}} = \min \{ \psi_i \}, \) with \( \lambda_i = \beta_i \lambda_1 \chi_i \Omega_L \) the average channel gain of the \( i \)-th SM conditioned on \( \chi_i, \ T_0 = \frac{m_i \gamma_0}{\nu_0 \lambda_i}, \ \Gamma (a) = \int_0^\infty t^{a-1} e^{-t} dt \) is the Gamma function,
\[
G_{m,n}^{p,q} \left( a_1, \ldots, a_p \mid b_1, \ldots, b_q \right) z \]
the Meijer G-function, and the coefficients \( \delta_k \) can be obtained recursively using the formula
\[
\delta_0 = 1,
\]
\[
\delta_{k+1} = \frac{1}{k+1} \sum_{r=1}^{k+1} m_L \left( 1 - \frac{\psi_{\text{min}}}{\psi_i} \right)^r \delta_{k+1-r}, \ k = 0, 1, 2, \ldots
\]

**Proof:** For convenience, the proof is presented in Appendix A.
\[\blacksquare\]
We note that the results in Theorem 1 provide an analytical closed-form of the outage probability conditioned on \( \chi \). Below we provide insights as to how Theorem 1 could be used to devise performance of our communication system in various particular cases. For instance, by setting \( p_i = 0, \forall i \in \{1, \ldots, K\} \), Theorem 1 captures a time division multiple access (TDMA) scheme when only one transmitter is allowed to communicate at time. In this case, the outage probability in (8) simplifies to:

\[
P_{\text{out}}(\gamma_T, \nu_0 | \chi) = 1 - \frac{\Gamma(m_L M, T_0)}{\Gamma(m_L M)} = \frac{\gamma (m_L M, T_0)}{\Gamma(m_L M)}
\]

(10)

where \( \gamma (s, x) = \int_0^x t^{s-1} e^{-t} \, dt \) is the lower incomplete Gamma function. On the other extreme, Theorem 1 also captures the case of fully loaded system when all the transmitters are communicating at the same time, by setting \( p_i = 1, \forall i \in \{1, \ldots, K\} \). In absence of shadowing and if we assume a receiver with one single antennas, we obtain the same result as in [24]. However, when \( m_L M \) is not an integer, it is difficult to find a closed-form expression of the (unconditioned) outage probability \( P_{\text{out}}(\gamma_T, \nu_0, \nu) \) by taking the expectation with respect to \( \chi \). Nevertheless, it can be accurately estimated through Monte Carlo simulations. Fortunately, a closed-expression of \( P_{\text{out}}(\gamma_T, \nu_0, \nu) \) may be obtained when \( m_L M \) is integer. This is formalized below.

**Theorem 2.** When \( m_L M \) is integer, the outage probability of SMS can be expressed by

\[
P_{\text{out}}(\gamma_T, \nu_0, \nu) = \frac{1}{\Gamma(m_L M)} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_0} \right)^M \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_0} \right)^k \frac{f_{\sigma^+, (Mm_L + k)}}{f_{\sigma^-, (Mm_L + k)}}
\]

(11)

where \( \delta_{t_i} \) is the Kronecker delta function defined as

\[
\delta_{t_i} = \begin{cases} 
1, & \text{if } t_i = 0 \\
0, & \text{if } t_i = 1,
\end{cases}
\]

(12)

\( \lambda_{i,L} = E_{\chi_i} (\lambda_{i,L}) = \beta_{i,L} \Omega L E_{\chi_i} (\chi_i) = \beta_{i,L} \Omega L \mathcal{E}^{\frac{2}{T} + \mu_s} \) is the average channel gain of the \( i^{th} \) SM, \( f_{\sigma^+, (x)} = e^{\frac{1}{T} \sigma^2 x (x+1)} \), and \( f_{\sigma^-, (x)} = e^{\frac{1}{T} \sigma^2 x (x-1)} \).
Proof: The proof is presented in Appendix B.

In absence of shadowing and if we assume that all transmitters are active with the same probability $p_i = 1$, $\forall i \in \{0, \ldots, K\}$, we obtain the same result as [25] for the outage probability. Next, we study the high SNR regime of the SM of interest, and derive an asymptotic expression for the outage probability, which enables the characterization of the achievable diversity order. Specifically, we characterize the two key performance parameters dictating the outage probability in the high SNR regime, i.e., the diversity gain $G_d$ and the array gain $G_a(\gamma)$ defined by [20]

$$P_{\text{out}}^\infty(\gamma, \nu_0, \nu) = G_a(\gamma, \nu) \nu_0^{-G_d} + O\left(\nu_0^{-(G_d+1)}\right),$$

where $O\left(\nu_0^{-(G_d+1)}\right)$ is a function of $\nu_0$ such that $\left|\frac{O\left(\nu_0^{-(G_d+1)}\right)}{\nu_0^{-(G_d+1)}}\right| \leq M_0$, for some positive $M_0$.

Corollary 1. In the high SNR regime, i.e., $\nu_0 \to \infty$, the outage probability of the system can be expressed as

$$P_{\text{out}}^\infty(\gamma, \nu_0, \nu) = G_a(\gamma, \nu) \nu_0^{-G_d} + O\left(\nu_0^{-(G_d+1)}\right),$$

where

$$G_d = m_L M,$$

$$G_a(\gamma, \nu) = E\xi\left(\frac{\nu_0^{-\nu_0^{-G_d+1}}}{\Gamma(m_L M + 1)} \prod_{i=1}^{K} (1 - p_i) + \sum_{S \subseteq E} \left[\prod_{i \in S, \ l \in E \setminus S} p_i (1 - p_i) \right] \right)$$

$$\times \prod_{i \in S} \left(\psi_{\min}\right)^{-\nu_0^{-G_d+1}} \sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(N_0 - k + 1, m_L M + k) \Gamma(-m_L M)} G_{2,1}^{1,2}\left(-N_0, m_L^{-k+1, m_L M+1} \mid \psi_{\min}\right),$$

and $E\xi(g(\xi))$ denotes the expectation of $g(\xi)$ with respect to $\xi$.

Proof: The proof is presented in Appendix C.

Corollary 1 presents the asymptotic expression for the outage probability. It indicates that SMS achieves a diversity gain of $M m_L$. It depends only on the number of receive antennas at the gateway and the Nakagami-fading parameter of the reference transmitter. By increasing one of those parameters, we increase the diversity gain and thus ameliorate the reliability of the system. Note that $G_a(\gamma, \nu) > 0$ since the last product in (16) is the pdf of sum of gamma variate.
IV. Outage performance with Blockage

Figure 2. The SMS configuration. Humans are modeled as circular cylinders of diameter $D$ and height $H$ within an enclosed space, with dimension $L$, $W$ and $H_a$.

In this section, we study the effect of the blockage on the performance of our system. The potential source of blockage for mmWave in indoor environments are humans and/or concrete structures, for instance. We model them as cylinders with a certain height, $H$, and the base diameter $D$ [26]. Differently from [15], [16], [19] that consider cubes-based blocking model with a fixed height, ball-based blocking model with a fixed radius, or 3D cylinders with fixed heights and diameters, we consider in our model that both $H$ and $D$ are RVs. We model human bodies as cylinders with different sizes, because we can found in the indoor environment kids, women and men. The distribution of height is Normal with mean $\mu_H$ and variance $\sigma_H^2$, i.e., $(H \sim \mathcal{N}(\mu_H, \sigma_H))$ [27]. The RV $D$ is assumed to be uniformly-distributed between $d_{min}$ and $d_{max}$. We use stochastic geometry to model these blockages. As $D$ is a RV, the centers of cylinder bases follow a Matern hard-core point process (MHCP) of type I [28], [29] on the plane with the intensity $\lambda_{b1}$ [30]. This process is appealing to ensure that the locations of blockers do not overlap. However, due to its complexity, MHCP is commonly substituting by the more tractable PPP with intensity $\lambda_0 \geq \lambda_{b1}$ [30]. In this section, we consider that each SM is located at certain height $H_i$ above the ground and the gateway located at the height $H_R$ as illustrated in Fig. 2 with $H_i$ and $H_R \in [0, H_a]$, where $H_a$ is the height of the ceiling. As a practical assumption, all $H_i$ are smaller than $H_R$, i.e., $H_i \leq H_R$. Our objective is to derive the system outage probability. As the latter depends on whether the SM is in LOS or in NLOS with the gateway, the attenuation
power-law exponent and the fading parameter of the $i^{th}$ SM have 2 possible values as follows

$$
\begin{align*}
\alpha_L \text{ w.p. } P_{\text{LOS}} (d_i) &; \quad m_L \text{ w.p. } P_{\text{LOS}} (d_i) \\
\alpha_N \text{ w.p. } 1 - P_{\text{LOS}} (d_i) &; \quad m_N \text{ w.p. } 1 - P_{\text{LOS}} (d_i)
\end{align*}
$$

(17)

where $\alpha_L$ is the attenuation power-law exponent for the LOS and $\alpha_N$ for NLOS, $m_L$ is the Nakagami factor for the LOS and $m_N$ for NLOS. We first evaluate the probability of LOS below.

**Lemma 1.** When the network region is rectangular and blockages have diameter $D \sim \mathcal{U} (d_{\text{min}}, d_{\text{max}})$ with height $H \sim \mathcal{N} (\mu_H, \sigma_H)$, the probability that the $i^{th}$ SM at distance $d_i$ from the gateway is in LOS is

$$
P_{\text{LOS}} (d_i) = \begin{cases} 
\frac{1}{2} (Q (d_{\text{max}}, d_{\text{min}}) \lambda_0 d_{\text{max}}) \left(1 - \frac{g_i (d_i) - g_i (0)}{d_i} \right) & \text{for } H_R \neq H_i \\
\frac{1}{2} (Q (d_{\text{max}}, d_{\text{min}}) \lambda_0 d_{\text{max}}) \left(1 - \text{erf} \left( \frac{H_R - \mu_H}{\sigma_H \sqrt{2}} \right) \right) & \text{otherwise}
\end{cases}
$$

(18)

where $Q (d_{\text{max}}, d_{\text{min}}) = \frac{d_{\text{min}}}{d_{\text{max}}} + \frac{d_{\text{max}} - d_{\text{min}}}{d_{\text{max}} (d_{\text{max}} - d_{\text{min}})}$, $g_i (x) = \frac{b_i \text{erf} (b_i + a_i x) + x \text{erf} (b_i + a_i x)}{a_i \sqrt{\pi}}$, with $a_i = \frac{H_R - H_i}{H_R \sigma_H \sqrt{2}}, b_i = \frac{H_i - \mu_H}{\sigma_H \sqrt{2}}$, and erf(.) is the error function.

**Proof:** For convenience, the proof is presented in Appendix D. \hfill \blacksquare

Now, we are ready to give the outage probability accounting for the blockage.

**Corollary 2.** In the presence of blockages, the outage probability of the smart metering system is equal to

$$
P_{\text{out}} (\gamma_T, \nu_0, \nu) = P_{\text{LOS}} (d_0) P_{\text{out}, L} (\gamma_T, \nu_0, \nu) + (1 - P_{\text{LOS}} (d_0)) P_{\text{out}, N} (\gamma_T, \nu_0, \nu),
$$

(19)

where $P_{\text{out}, L} (\gamma_T, \nu_0, \nu)$ and $P_{\text{out}, N} (\gamma_T, \nu_0, \nu)$ are the outage probabilities of SM of interest
when it is in LOS or NLOS with the gateway, respectively, given by

\[ P_{\text{out},L}(\gamma_T, \nu_0, \mathbf{\nu}) = \frac{1}{\Gamma(m_L M)} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_{0,L}} \right)^{M m_L} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (M m_L + k)} \left( \frac{\gamma_T m_L}{\nu_0 \lambda_{0,L}} \right)^k f_{\sigma,+} (M m_L + k) \]

\[ \times \sum_{t=0}^{M m_L + k} \binom{M m_L + k}{t} t! \sum_{\sum_{i=1}^{K} t_i = t} \prod_{i=1}^{K} (1 - p_i) \delta_{t_i} + \frac{p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L)}{t_i! \Gamma(m_L)} \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i} f_{\sigma,-} (t_i) \]

\[ + \frac{p_i (1 - P_{\text{LOS}}(d_i)) \Gamma(t_i + m_N)}{t_i! \Gamma(m_N)} \left( \frac{\nu_i \lambda_{i,N}}{m_N} \right)^{t_i} f_{\sigma,-} (t_i) \]

(20)

\[ P_{\text{out},N}(\gamma_T, \nu_0, \mathbf{\nu}) = \frac{1}{\Gamma(m_N M)} \left( \frac{\gamma_T m_N}{\nu_0 \lambda_{0,N}} \right)^{M m_N} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (M m_N + k)} \left( \frac{\gamma_T m_N}{\nu_0 \lambda_{0,N}} \right)^k f_{\sigma,+} (M m_N + k) \]

\[ \times \sum_{t=0}^{M m_N + k} \binom{M m_N + k}{t} t! \sum_{\sum_{i=1}^{K} t_i = t} \prod_{i=1}^{K} (1 - p_i) \delta_{t_i} + \frac{p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L)}{t_i! \Gamma(m_L)} \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i} f_{\sigma,-} (t_i) \]

\[ + \frac{p_i (1 - P_{\text{LOS}}(d_i)) \Gamma(t_i + m_N)}{t_i! \Gamma(m_N)} \left( \frac{\nu_i \lambda_{i,N}}{m_N} \right)^{t_i} f_{\sigma,-} (t_i) \]

(21)

with \( \lambda_{i,L} \) and \( \lambda_{i,N} \) are the average channel gains of the \( i \)th SM in cases of LOS and NLOS, respectively.

**Proof:** For convenience, the proof is presented in Appendix E. \( \square \)

Next, we study the high SNR regime of the SM of interest, and derive an asymptotic expression for the outage probability in the presence of blockages, which enables the characterization of the achievable diversity order.

**Corollary 3.** In the high SNR regime, the outage probability of the system, in presence of blockages, can be expressed by

\[ P_{\text{out}}^{\infty}(\gamma_T, \nu_0, \mathbf{\nu}) = (1 - P_{\text{LOS}}(d_0)) P_{\text{out},N}^{\infty}(\gamma_T, \nu_0, \mathbf{\nu}) + O \left( \nu_0^{-(m_N M + 1)} \right), \]  

(22)
where
\[
P_{\text{out},N}(\gamma_T, \nu_0, \nu) \cong \left( \frac{\gamma_T m_N}{\nu_0 \lambda_0^N} \right)^{Mm_N} \frac{f_{\sigma_s, +}(Mm_N)}{\Gamma(m_N M + 1)}
\]
\[
\times \sum_{t=0}^{Mm_N} \binom{Mm_N}{t} t! \sum_{i=1}^{K} \prod_{i=1}^{K} \left[ (1 - p_i) \delta_{t_i} + \frac{p_i P_{\text{LOS}}(d_i) \Gamma(t_i + m_L)}{t_i ! \Gamma(m_L)} \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i} f_{\sigma_s,-}(t_i) \right]
\]
\[
+ \frac{p_i (1 - P_{\text{LOS}}(d_i)) \Gamma(t_i + m_N)}{t_i ! \Gamma(m_N)} \left( \frac{\nu_i \lambda_{i,N}}{m_N} \right)^{t_i} f_{\sigma_s,-}(t_i)
\]
\[
(23)
\]

**Proof:** The proof is presented in Appendix F.

Note that, in the high average SNR regime, \( P_{\text{out},L}(\gamma_T, \nu_0, \nu) \ll P_{\text{out},N}(\gamma_T, \nu_0, \nu) \). In this case, the achievable diversity order is equal to \( Mm_N \), as indicated by (23).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_0 )</td>
<td>6 m</td>
<td>Distance between the gateway and the SM of interest</td>
</tr>
<tr>
<td>( d_{\text{ref}} )</td>
<td>1 m</td>
<td>The reference distance</td>
</tr>
<tr>
<td>( m_L )</td>
<td>4</td>
<td>Nakagami fading parameter for LOS link</td>
</tr>
<tr>
<td>( m_N )</td>
<td>2</td>
<td>Nakagami fading parameter for NLOS link</td>
</tr>
<tr>
<td>( \Omega_L = \Omega_N )</td>
<td>1</td>
<td>Nakagami spread parameters for LOS and NLOS link are equal to one</td>
</tr>
<tr>
<td>( \alpha_L )</td>
<td>2</td>
<td>Path-loss exponent for LOS link</td>
</tr>
<tr>
<td>( \alpha_N )</td>
<td>4</td>
<td>Path-loss exponent for NLOS link</td>
</tr>
<tr>
<td>( K )</td>
<td>44</td>
<td>Number of interfering SMs</td>
</tr>
<tr>
<td>( p_i, 1 \leq i \leq K )</td>
<td>( p_i )</td>
<td>All the interfering SMs transmit with the same probability of success ( p_i )</td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>0</td>
<td>The mean of shadowing factor ( \eta_s \sim N(\mu_s, \sigma_s) )</td>
</tr>
<tr>
<td>( H )</td>
<td>( N(1.7m, 0.1m) )</td>
<td>Height of blocker, ( N(\mu_H, \sigma_H) )</td>
</tr>
<tr>
<td>( D )</td>
<td>( U(0.2m, 0.8m) )</td>
<td>Diameter of blocker, ( U(\text{min}, \text{max}) )</td>
</tr>
<tr>
<td>( \lambda_b )</td>
<td>0.3 blockers/m²</td>
<td>Intensity of blockers</td>
</tr>
<tr>
<td>( H_i, 0 \leq i \leq K )</td>
<td>1.3 m</td>
<td>All the SMs has the same height</td>
</tr>
<tr>
<td>( H_R, 0 \leq i \leq K )</td>
<td>4 m</td>
<td>The height of the gateway</td>
</tr>
</tbody>
</table>
V. NUMERICAL RESULTS

In this section, we present numerical results for the outage probability accounting for the blockages. The SMs are located in a uniform $5 \times 9$ rectangular grid as shown in Fig. 3. The number of interferers $K = 44$. The SM of interest is located at distance $d_0 = 6$ m from the gateway. The distances of the 44 interferers to the gateway are given in Fig. 3. We choose the distances between the gateway and SMs so small because they are confined in an indoor environment, i.e., the gateway and the SMs are located in the same floor of the building. The latter is located at the centre of the rectangle. The parameters used in this section are summarized in Table I.

Figure 4 shows the outage probability as function of the transmit SNR $\nu_0$ of the reference SM.

![Figure 3. The locations of SMs in a uniform rectangular grid of size 5×9.](image)

This figure illustrates that Monte Carlo simulations matches the analytical expression in (19). The asymptotic expression given in (22) provides accurate prediction of the outage probability in the high SNR regime. As illustrated in Fig. 4, the outage probability in the absence of blockages (20) is dominated by the outage probability with blockages in the high average SNR regime, thus confirming the result in corollary 3. It can be seen from Fig. 4 that increasing $M$, improves the diversity gain ($G_d = Mm_N$).

The dependence of $P_{\text{out}}(\gamma_T, \nu_0, \nu)$ on different transmission probabilities $p_t$ is shown in Fig. 5. It can be seen in Fig. 5 that the outage probability increases with $p_t$, for a given threshold.
Figure 4. Outage probability vs $\nu_0$ for different number of $M$ using the analytic expression (19), (22) and (20) ($\nu_i = 20$ dB, $\forall i \in \{1, \ldots, K\}, \sigma_\epsilon^2 = 0$ dB, $\gamma_T = 10$ dB, $p_t = 0.1$).

Figure 5. Outage probability vs SINR threshold $\gamma_T$ for different transmission probability values $p_t$ ($\nu_i = 20$ dB, $\forall i \in \{0, \ldots, K\}, M = 4, \sigma_\epsilon^2 = 0$ dB).

The coverage probability, i.e., $P_c(\gamma_T, \nu_0, \nu) = 1 - P_{\text{out}}(\gamma_T, \nu_0, \nu)$, is plotted versus the threshold $\gamma_T$, for $K = 4$ and $K = 44$, with and without blockages in Fig. 6. An interesting result is that although the blockage deteriorates the diversity gain (Corollary 3 and Fig. 4), it may improve the coverage at finite SNR, if the number of interferers is low ($K = 4$ in Fig. 6). This result can be explained by the fact that blockages are more likely to attenuate the effect of the interferers. This is a good feature of mmWave frequencies over lower frequencies as the
The former are much more sensitive to blockages than the latter. As illustrated in Fig.6, when the coverage probability equals to 0.8, the presence of blockages with 4 interferers provides a 5.5 dB gain over the absence of blockages. Another observation is that when the number of interferers is very large, the coverage probability converges to the case with no blockages, i.e., increasing $K$ will close the gap. This can be explained by the fact that increasing the number of interferers with a fixed blockage density is equivalent to decreasing the blockage density.

Figure 7 shows the variation of the coverage probability in function with $\lambda_b$, the blockage

Figure 6. Coverage probability vs $\gamma_T$ with/without blockages $(\nu_i = 20 \text{ dB}, \forall i \in \{0, \ldots, K\}, M = 1, p_i = 0.1, \sigma_i^2 = 0 \text{ dB})$.

Figure 7. Coverage probability versus $\lambda_b$ for different values of SINR threshold $\gamma_T$, using the analytic expression (Corollary 2) $(\nu_i = 20 \text{ dB}, \forall i \in \{0, \ldots, K\}, M = 1, p_i = 0.1, \sigma_i^2 = 0 \text{ dB})$.
density. It is seen that as $\lambda_b$ increases, the coverage probability improves rapidly in the case when $\gamma_T = 10$ dB. This supports our interpretation of Fig. 6 that high density of blockages improves the probability of coverage.

VI. CONCLUSION

In this paper, we have studied the outage probability of mmWave SMS operating in an indoor environment. We incorporated path-loss, shadowing, Nakagami fading into our channel. We assumed Aloha medium-access protocol. We proposed a framework to model random blockages in an indoor environment using concepts and tools from stochastic geometry. The key idea is to model humans as random cylinders whose centers follow PPP with arbitrary heights and diameters. Based on this geometric blockage model, we derived the probability of outage and highlighted the corresponding diversity gain. The model captures the dependence of probability of LOS on the distances. Our framework also highlights the effect of the medium access protocol (Aloha) on the outage probability. Perhaps surprisingly, the developed analysis indicates that blockages change the behaviour of SMS in an important way in the sense that it may improve the coverage probability at finite SNR.

APPENDIX A

PROOF OF THEOREM 1

We define the conditioned outage probability $P_{\text{out}}(\gamma_T, \nu_0, \nu|\chi)$ as the probability that the received SINR is below a given threshold $\gamma_T$ for a given $\nu_0$ and $\nu = [\nu_1, \ldots, \nu_K]$ conditioned on $\chi = [\chi_0 = 10^{\nu_0}, \chi_1, \ldots, \chi_K = 10^{\nu_K}]$. Therefore, we can write

$$P_{\text{out}}(\gamma_T, \nu_0, \nu|\chi) = Pr\left(\gamma(\nu_0, \nu) = \frac{\nu_0 \zeta_0}{\sum_{i=1}^{K} I_i^2 \nu_i \zeta_i + 1} \leq \gamma_T | \chi \right).$$

We define $R \triangleq \gamma_T^{-1} \nu_0 \zeta_0$, $Z_i \triangleq I_i^2 \nu_i \zeta_i \triangleq \sum_{i=1}^{K} Z_i$. The outage probability can be expressed as follows

$$P_{\text{out}}(\gamma_T, \nu_0, \nu|\chi) = 1 - Pr\left(R \geq Z + 1 | \chi \right) \triangleq 1 - P(\gamma_T, \nu_0, \nu|\chi).$$

(25)
Conditioned on $\chi$, let $f_Z(z)$ denotes the probability density function (pdf) of $Z$ and $f_R(r)$ denotes the pdf of $R$. Using these definitions, $P(\gamma_T, \nu_0, \nu|\chi)$ may be expressed as

$$P(\gamma_T, \nu_0, \nu|\chi) = \int_0^\infty f_Z(z) \left( \int_{(z+1)}^\infty f_R(r) \, dr \right) \, dz. \quad (26)$$

When the transmission is made over Nakagami-m fading channels, it is possible to show [31] that $b_i (1 \leq i \leq K)$ and $b_0$ follow a Gamma distribution with a shape and scale parameters $(m_L, \frac{\nu_0}{m_L})$, $(m_L \cdot M, \frac{\nu_0}{m_L})$, respectively. Therefore, $\zeta_i (1 \leq i \leq K)$ and $\zeta_0$ follow a Gamma distribution with shape and scale parameters $(m_L, \frac{\nu_0}{m_L})$, $(m_L \cdot M, \frac{\nu_0}{m_L})$, respectively, where $\lambda_i = \beta_i \chi \Omega_L$ is the average channel gain of the $i^{th}$ SM conditioned on $\chi_i$ in the LOS case. Hence, $R$ is a Gamma RV with parameters $(m_L \cdot M, \frac{\gamma_i \nu_0 \lambda_i^2}{m_L})$. After using the simple transformation $u = \frac{\gamma_i \nu_0 \lambda_i^2}{m_L} r$, the inner integral in (26) becomes:

$$\int_{(z+1)}^\infty f_R(r) \, dr = \frac{\Gamma(m_L \cdot M, (z+1) T)}{\Gamma(m_L \cdot M)}, \quad (27)$$

where $T = \frac{\gamma_i \nu_0 \lambda_i^2}{m_L}$ and $\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} \, dt$ is the upper incomplete Gamma function.

We have $Z_i = I_i^2 \nu_0 \zeta_i$, where $\zeta_i$ is a Gamma RV and $I_i$ is a Bernoulli RV. Let the RV $N_i = I_i^2$. $N_i$ is also a Bernoulli RV with the same probability of success $p_i$ as $I_i$. So, the pdf of $Z_i$ is

$$f_{Z_i}(z_i) = (1 - p_i) \delta(z_i) + f_{B_i}(z_i), \quad (28)$$

where $\delta(z)$ is the delta function and $f_{B_i}(z_i)$ is the pdf of the Gamma RV $(B_i = \nu_0 \zeta_i)$ with shape and scale parameters, $(m_L)$ and $(\frac{\nu_0 \lambda_i}{m_L})$, respectively.

We assume that $\{Z_i\}_{i=1}^K$ are independent for a given $\chi$. So, $f_Z(z)$ can be written as $\prod_{i=1}^K f_{Z_i}(z_i)$, where the product is in term of convolution. After some manipulation, the pdf of $Z$ is given by the following expression

$$f_Z(z) = \prod_{i=1}^K (1 - p_i) \delta(z) + \sum_{S \subseteq E} \left( \prod_{i \in S, \, \ell \in E \setminus S} p_i (1 - p_i) \right) \left( \prod_{i \in S} f_{B_i}(z) \right). \quad (29)$$

The summation $\left( \sum_{S \subseteq E} \right)$ is taken over all subsets $S$ such that $S \subseteq E = \{1, \ldots, K\}$, and $\prod_{i \in S} f_{B_i}(z)$ is the product in term of convolution. This product can be seen as the pdf of the sum of gamma variate. This pdf can be obtained using Moschopoulos theorem [32], as follows

$$\prod_{i \in S} f_{B_i}(z) = \prod_{i \in S} \left( \frac{\psi_{\min}}{\psi_i} \right)^{m_L} \sum_{k=0}^{\infty} \frac{\delta_k z^{NS} m_L + k - 1}{\psi_{\min}^{NS} m_L + k} e^{-\frac{z}{\psi_{\min}}} \frac{1}{\Gamma(N_S m_L + k)} \quad (30)$$
where \( N_S = \text{card}(S) \) is the number of elements in subset \( S \), \( \psi_i = \frac{\nu_i \zeta_i}{m_L} \), \( i \in S \), \( \psi_{\text{min}} = \min_{i \in S}(\psi_i) \), and the coefficients \( \delta_k \) can be obtained recursively using the formula

\[
\begin{cases}
\delta_0 = 1, \\
\delta_{k+1} = \frac{1}{k+1} \sum_{r=1}^{k+1} \left[ \sum_{i \in S} m_L \left( 1 - \frac{\psi_{\text{min}}}{\psi_i} \right)^r \right] \delta_{k+1-r}, \ k = 0, 1, 2, \ldots
\end{cases}
\]  

(31)

We use [33, eq. (8.354.2)] to express (27) and substituting the result in (26), we obtain the following expression

\[
P(\gamma_T, \nu_0, \nu| \chi) = 1 - \sum_{j=0}^{\infty} \frac{(-1)^j T^{m_L M+j}}{j!(m_L M + j)} \int_0^\infty (z + 1)^{m_L M+j} f_Z(z) \, dz
\]  

(32)

Substituting (30) into (29), the inner integral in (32) is equal to

\[
\int_0^\infty (z + 1)^{m_L M+j} f_Z(z) \, dz = \prod_{i=1}^K (1-p_i) + \sum_{S \subseteq E} \left[ \prod_{i \in S, l \in E \setminus S} p_i (1-p_l) \right] \times \left[ \prod_{i \in S} \left( \frac{\psi_{\text{min}}}{\psi_i} \right)^{m_L} \sum_{k=0}^{\infty} \psi_{\text{min}}^{N_S m_L+k} \delta_k \Gamma(N_S m_L+k) \int_0^\infty (z + 1)^{m_L M+j} z^{N_S m_L+k-1} e^{-\frac{z}{\psi_{\text{min}}}} \, dz \right].
\]  

(33)

We use [34, eq. (07.34.03.0271.01)] and [33, eq. (7.813.1)] to solve the inner integral in (33) as follows

\[
\int_0^\infty (z + 1)^{m_L M+j} z^{N_S m_L+k-1} e^{-\frac{z}{\psi_{\text{min}}}} \, dz = \psi_{\text{min}}^{N_S m_L+k} \Gamma^{-1}(-m_L M - j) \Gamma_{2,1}^{1,2} \left( -N_S m_L-k+1, m_L M+j+1 \bigg| \psi_{\text{min}} \right)
\]  

(34)

We substitute (34) into (33) and (33) into (32). In the final step, we substitute (32) into (25) to obtain the final result.

**APPENDIX B**

**PROOF OF THEOREM 2**

In this section, we will use the same steps to obtain the conditioned outage probability \( P_{\text{out}}(\gamma_T, \nu_0, \nu| \chi) \) in theorem 1, but, we will define \( f_Z(z) \) which is the joint pdf of \( Z = [Z_1, Z_2, \ldots, Z_K] \), where \( Z_i = I_i^2 \nu_i \zeta_i \). After these modifications, (26) becomes

\[
P(\gamma_T, \nu_0, \nu| \chi) = \int_{R^K_+} \cdots \int_{R^K_+} f_Z(z) \left( \int_{\sum_{i=1}^K z_i+1}^\infty f_R(r) \, dr \right) \, dz.
\]  

(35)
We use [33, eq. (8.354.2)] to express the upper incomplete Gamma function in (27) as follows:

\[
\int_0^\infty f_R(r)dr = 1 - \frac{1}{\Gamma(mLM)} \sum_{k=0}^\infty \frac{(-1)^k T^{Mm_L+k}}{k! (Mm_L + k)} \left( 1 + \sum_{i=1}^K \frac{z_i}{Mm_L + k} \right)^t. 
\]  

(36)

We can use the binomial theorem since \((Mm_L + k)\) is a positive integer to develop the inner expression as follows

\[
\left( 1 + \sum_{i=1}^K \frac{z_i}{Mm_L + k} \right)^t = \sum_{t=0}^{Mm_L+k} \binom{Mm_L+k}{t} \left( \sum_{i=1}^K \frac{z_i}{Mm_L + k} \right)^t. 
\]  

(37)

Using multinomial expansion, we can obtain

\[
\left( \sum_{i=1}^K \frac{z_i}{Mm_L + k} \right)^t = t! \sum_{\sum_{i=1}^K t_i = t} \left( \prod_{i=1}^K \frac{z_i^{t_i}}{t_i!} \right) 
\]  

(38)

where the summation in (38) is taken over all the combinations such that \(\sum_{i=1}^K t_i = t, \ t_i \in \mathbb{N}\).

We assume that \(\{Z_i\}_{i=1}^K\) are independent for a given \(\chi\). So, \(f_Z(z)\) can be written as \(\prod_{i=1}^K f_{Z_i}(z_i)\). Using this assumption, replacing \(T\) by \(\frac{\gamma m_L}{\nu_0 \lambda_0}\), and substituting (38) and (37) into (36), we obtain a new reformulation of (35) as

\[
P(\gamma_T, \nu_0, \nu | \chi) = 1 - \frac{1}{\Gamma(mLM)} \sum_{k=0}^\infty \frac{(-1)^k \left( \frac{\gamma m_L}{\nu_0 \lambda_0} \right)^{Mm_L+k}}{k! (Mm_L + k)} \sum_{t=0}^{Mm_L+k} \binom{Mm_L+k}{t} t! \prod_{i=1}^K \int_0^\infty \frac{z_i^{t_i}}{t_i!} f_{Z_i}(z_i) dz_i. 
\]  

(39)

Using (28), the integral in (39) will be expressed as

\[
G_{(p_i,t_i,m_L,\nu_i)}(\lambda_{i,L}) = (1 - p_i) \delta_{t_i} + p_i \Gamma(t_i + m_L) \frac{\nu_i \lambda_{i,L}^{t_i}}{t_i! \Gamma(m_L)} \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i}, 
\]  

(40)

where \(\delta_{t_i}\) is the Kronecker delta function defined as

\[
\delta_{t_i} = \begin{cases} 
1, & \text{if } t_i = 0 \\
0, & \text{if } t_i = 1.
\end{cases} 
\]  

(41)
Substituting (40) into (39), the conditioned outage probability becomes

\[
P_{\text{out}} (\gamma_T, \nu_0, \nu | \chi) = \frac{1}{\Gamma (m_L M)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (M m_L + k)^{M m_L + k}} \sum_{t=0}^{M m_L + k} \left( \begin{array}{c} M m_L + k \\ t \end{array} \right) t!
\]

\[
\times \sum_{\sum_{i=1}^{K} t_i = t} \prod_{i=1}^{K} G_{(p_i, t_i, m_L, \nu_i)} (\lambda_{i,L}^c).
\]

The next step, we derive the conditioned outage probability \( P_{\text{out}} (\gamma_T, \nu_0, \chi | \chi_0) \), by taking an expectation with respect to \( \chi_s = [\chi_1, ..., \chi_K] \). We assume that \( \{\chi_i\}_{i=1}^{K} \) are independent. Therefore, this leads us to evaluate \( E_{\chi_i} [G_{(p_i, t_i, m_L, \nu_i)} (\lambda_{i,L}^c)] \),

\[
E_{\chi_i} [G_{(p_i, t_i, m_L, \nu_i)} (\lambda_{i,L}^c)] = \int_{0}^{\infty} f_{\chi_i}^c (\lambda_{i,L}) G_{(p_i, t_i, m_L, \nu_i)} (\lambda_{i,L}^c) d\lambda_{i,L}^c,
\]

where

\[
f_{\chi_i}^c (\lambda_{i,L}) = \frac{1}{\lambda_{i,L}^c \beta_{i,L} \Omega_L \sqrt{2\pi}} e^{-\frac{(\ln \lambda_{i,L}^c - \mu_s)^2}{2\sigma_s^2}}.
\]

The pdf of \( \lambda_{i,L}^c \) depends on the pdf of the Lognormal RV \( \chi_i = 10^{m_i} \). Using [33, eq. (3.323.2)], (44) and by making a change of variable \( r_i = \ln \frac{\lambda_{i,L}^c}{\beta_{i,L} \Omega_L} \) to calculate (43), we obtain the expression of \( P_{\text{out}} (\gamma_T, \nu_0, \nu | \chi_0) \).

In the final step, we derive \( P_{\text{out}} (\gamma_T, \nu_0, \nu) \) by taking the expectation with respect to \( \chi_0 \), and considering the average channel gain of the \( i^{th} \) SM \( \lambda_{i,L} = E_{\chi_i} (\lambda_{i,L}^c) = \beta_{i,L} \Omega_L E_{\chi_i} (\chi_i) = \beta_{i,L} \Omega_L e^{\frac{(\mu_s + 1)}{2} + \mu_s} \).

APPENDIX C

PROOF OF COROLLARY 1

In the high average SNR regime, \( \nu_0 \to \infty \) and thus \( T \to 0 \). Using the expansion of regularized Gamma function at 0 [34, eq. (06.08.06.0004.02)], (27) is expressed as follows in the high average SNR

\[
\frac{\Gamma (m_L M, (z + 1) T)}{\Gamma (m_L M)} = 1 - \frac{(z + 1)^{m_L M} T^{m_L M}}{\Gamma (m_L M + 1)} (1 + O(T^{m_L + 1}))
\]

\[
= 1 - \frac{(z + 1)^{m_L M} T^{m_L M}}{\Gamma (m_L M + 1)} + O(T^{m_L + 1}).
\]

By properties of the Gamma function, it is clear that

\[
\left| f_Z (z) \frac{\Gamma (m_L M, (z + 1) T)}{\Gamma (m_L M)} \right| \leq f_Z (z), \forall z \in [0, \infty],
\]

(46)
and $f_Z(z) : [0, \infty] \to [0, \infty]$ is integrable as $\int_0^\infty f_Z(z)dz = 1$. Therefore, we can use Lebesgue dominated convergence in (26). Using this theorem, (45), (29), and taking the expectation with respect to $\chi = [\chi_0 = 10^{\eta_0}, \ldots, \chi_K = 10^{\eta_K}]$, we obtain the formula given in corollary 1.

**APPENDIX D**

**PROOF OF LEMMA 1**

Figure 8. Top view of the blocking area.

Figure 9. The interfering SM locating at $d_i$ has a height of $H_i$, while the gateway has a height of $H_R$. Not all blockages which cross $OQ$ blockage the actual propagation path $AB$ in $R^3$, such as blockage (a) in the figure. If a blockage intersecting $OQ$ at a point $r$ away from the gateway effectively blocks $AB$ if and only if its height is larger than $r$ as blockage (b) in the figure.

The distribution of the number of blockages $L$ is required between each SM and gateway to derive probability of LOS. We show that $L$ is a Poisson distributed RV. We assume that teh SMs and the gateway are infinitesimal compared to the blockages (humans). They are represented as points as illustrated in Fig. 8. The centers of blockage are drawn from PPP with parameter
\( \lambda_b \) inside a rectangular-shaped area with width \( W \) and length \( L \). Consider a SM \( T_i \) located at distance \( d_i \) from the gateway. Its signal will be blocked if there is a blockage inside the subregion \( A \) of the rectangle. This subregion looks like in Fig. 8. Its width is bounded by \( d_{\text{max}} \), that is, the maximum width of blockers and its length is equal to \( d_i \). We call this subregion as the blocking zone of the \( i^{th} \) SM. From [35] which states that Poisson Law is preserved by thinning. Therefore, the number of blocker centers follow a Poisson distribution with average equals to \( \lambda_b d_{\text{max}} d_i \). However, not all the blockers inside \( A \) will block the signals from \( T_i \) to the gateway.

That is why, we define the following events:

(i) Event \( B_1 \) that the blocker’s radius is large enough to cross the LOS between the \( T_i \) and \( R_0 \).

(ii) Event \( C_1 \) that the blocker is high enough to block the LOS. A blockage will block the signal from \( T_i \) to \( R_0 \), if its radius and height are larger and longer enough to cross the LOS. Therefore, the average number of blockage \( L \) is equal to

\[
E(L) = \lambda_b d_{\text{max}} d_i Pr \{ B_1 \} Pr \{ C_1 \} \tag{47}
\]

The next step, we calculate the probability of each event. Consider a blockage intersecting the link \( OX \) at horizontal distance \( r \) away from \( X \) which the projection of gateway at the ground. As shown in Fig. 9, the human blocks the direction propagation path \( O'X' \), if its height \( H > H_i (r) \), where \( h(z) \) can be computed as follows

\[
H_i (r) = -\frac{H_i - H_R}{d_i} r + H_i, \quad r \in [0, d_i]. \tag{48}
\]

As the number of blockages \( L \) inside the blocking zone is Poisson RV, the intersections between the blockages and the link \( (OX) \) form a Poisson Process in \( (OX) \). Therefore, given that \( k \) humans intersect \( OX \), the \( k \) intersections are independently and uniformly distributed in the interval \( [0, d_i] \) [35, Definition 1.1.1]. Hence, given a blockage intersects \( OX \), the conditional probability that it blocks \( O'X' \) is

\[
Pr \{ C_1 \} = \int_{0}^{d_i} f_{R_i} (r) Pr [H > H_i (r)] dr = \int_{0}^{d_i} f_{R_i} (r) (1 - F_H (H_i (r))) dr \tag{49}
\]
where \( f_{R_i}(r) \) is the pdf of uniform distribution from 0 to \( d_i \), and \( F_H(x) \) is the cumulative distribution function (CDF) of the blocker’s height. Hence, we have

\[
f_{R_i}(r) = \begin{cases} \frac{1}{d_i}, & \text{if } r \in [0, d_i] \\ 0, & \text{otherwise} \end{cases}
\]  
(50)

Since \( H \sim N(\mu_H, \sigma_H) \), we have

\[
F_H(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x - \mu_H}{\sigma_H \sqrt{2}} \right) \right]
\]  
(51)

where \( \text{erf}(.) \) is the error function.

Substituting (51) and (50) into (49), we obtain

\[
\Pr\{C_1\} = \frac{1}{2} - \frac{1}{2d_i} \int_0^{d_i} \text{erf} \left( \frac{H_i(r) - \mu_H}{\sigma_H \sqrt{2}} \right) \, dr
\]  
(52)

Using [34, eqs. (006.25.21.0001.01)], the integral in (52) is expressed as follows

\[
\int_0^{d_i} \text{erf} \left( \frac{H_i(r) - \mu_H}{\sigma_H \sqrt{2}} \right) \, dr = \begin{cases} g_i(d_i) - g_i(0) & \text{for } H_i \neq H_R \\ d_i \text{ erf} \left( \frac{H_R - \mu_H}{\sigma_H \sqrt{2}} \right) & \text{otherwise} \end{cases}
\]  
(53)

where \( g_i(x) = \frac{b_i \text{erf}(b_i + a_i x)}{a_i} + x \text{erf}(b_i + a_i x) + e^{-\frac{(b_i + a_i x)^2}{a_i \pi}} \).

Substituting (53) into (52), we obtain the final result as

\[
\Pr\{C_1\} = \begin{cases} \frac{1}{2} - \frac{1}{2d_i} \left[ g_i(d_i) - g_i(0) \right] & \text{for } H_i \neq H_R \\ \frac{1}{2} - \frac{1}{2} \text{ erf} \left( \frac{H_R - \mu_H}{\sigma_H \sqrt{2}} \right) & \text{otherwise} \end{cases}
\]  
(54)

The probability of event \( B_1 \) is the probability that the radius of the blocker’s base is large enough to cross the LOS between \( T_i \) and \( R \). This means that the absolute value of the blocker’s \( y \) coordinate is greater than its radius, i.e. \(|y| > \frac{R}{2} \) as illustrated in Fig. 8. As the number of blocker centers in the blocking zone follows a PPP, the \( y \) coordinates of each center are independently and uniformly distributed on the interval \([\frac{d_{\min}}{2}, \frac{d_{\max}}{2}]\). Hence, the probability of event \( B_1 \) is equal to

\[
\Pr\{B_1\} = \int_{-\frac{d_{\max}}{2}}^{\frac{d_{\max}}{2}} f_Y(y) \Pr[|y| > Rd] \, dy
\]

\[
= \int_{-\frac{d_{\max}}{2}}^{\frac{d_{\max}}{2}} f_Y(y) (1 - F_{Rd}(|y|)) \, dy
\]  
(55)
where \( f_Y(y) \) is the pdf of uniform distribution from \(-\frac{d_{\max}}{2}\) to \(\frac{d_{\max}}{2}\), and \(F_{Rd}(rd)\) is the cumulative distribution function (CDF) of the blocker’s radius. Since blocker’s diameter \(D\) is uniformly distributed on the interval \(\left[\frac{d_{\min}}{2}, \frac{d_{\max}}{2}\right]\), its radius \(Rd = \frac{D}{2}\) is also uniformly distributed on the interval \(\left[\frac{d_{\min}}{2}, \frac{d_{\max}}{2}\right]\) and has the following cdf expression

\[
F_{Rd}(rd) = \begin{cases} 
0, & \text{for } rd < \frac{d_{\min}}{2} \\
\frac{2rd-d_{\min}}{d_{\max}-d_{\min}}, & \text{for } rd \in \left[\frac{d_{\min}}{2}, \frac{d_{\max}}{2}\right) \\
1, & \text{for } rd \geq \frac{d_{\max}}{2}
\end{cases}
\] (56)

\(f_Y(y)\) has the following expression

\[
f_Y(y) = \begin{cases} 
\frac{1}{d_{\max}}, & \text{for } y \in \left[\frac{-d_{\max}}{2}, \frac{d_{\max}}{2}\right] \\
0, & \text{otherwise}
\end{cases}
\] (57)

Substituting (57) and (56) into (55), we obtain

\[
Pr\{B_1\} = \frac{d_{\min}}{d_{\max}} + \frac{d_{\max}^2 + \left(\frac{d_{\min}}{2} - d_{\max}\right)d_{\min}}{d_{\max}(d_{\max} - d_{\min})}. 
\] (58)

Since \(L\) is a Poisson RV with average \(E(L) = \lambda_b d_i Pr\{B_1\} Pr\{C_1\}\), the probability that a link with length \(d_i\) admits LOS propagation, i.e., no blockages cross the link, is

\[
P_{LOS}(d_i) = Pr\{L = 0\} = e^{-E(L)} = e^{-\lambda_b d_i Pr\{B_1\} Pr\{C_1\}} \] (59)

Substituting (57) and (54) into (59), we obtain the expression of \(P_{LOS}(d_i)\).

**APPENDIX E**

**PROOF OF COROLLARY 2**

After defining the probability of LOS of each SM, the fading parameters and the attenuation power-law exponent have 2 possible values: \(m_L\) and \(\alpha_L\) for the LOS and \(m_N\) and \(m_N\) for the NLOS. For simplicity, we assume that \(m_L\) and \(m_N\) are integers as the outage probability will be more complicated if they are non integers. Hence, we follow the steps in theorem 2 to derive the outage probability in the presence of blockage. However, the main difference is that the path loss is now a discrete RV. After considering the probability of transmission of each transmitter \((p_0 = 1, \text{ the SM of interest is always transmitting})\), the path loss of the \(i^{th}\) SM is equal to

\[
\beta_{i,L} = \left(\frac{d_i}{d_{\text{ref}}}\right)^{-\alpha_L}, \text{ with probability } (1 - p_i) \\
\beta_{i,N} = \left(\frac{d_i}{d_{\text{ref}}}\right)^{-\alpha_N}, \text{ with probability } (p_i P_{LOS}(d_i)) \] (60)
Now, the RVs \( \{Z_i\}_{i=1}^K \) and \( R \) have the following pdf:

\[
f_R(r) = P_{\text{LOS}}(d_0) \ f_{C_{0,L}}(r) + (1 - P_{\text{LOS}}(d_0)) \ f_{C_{0,N}}(r),
\]

\[
f_{Z_i}(z_i) = (1 - p_i) \delta(z_i) + p_iP_{\text{LOS}}(d_i) \ f_{B_{i,L}}(z_i) + p_i (1 - P_{\text{LOS}}(d_i)) \ f_{B_{i,N}}(z_i),
\]

where \( f_{C_{0,L}}(\cdot), \ f_{C_{0,N}}(\cdot), \ f_{B_{i,L}}(\cdot), \) and \( f_{B_{i,N}}(\cdot) \) are the pdfs of the Gamma RVs which have the following distributions

\[
\begin{align*}
C_{0,L} &= \text{Gamma} \left( m_L M, \gamma_T^{-1} \nu_0 \frac{\lambda_{0,L}^c}{m_L} \right) ; \quad B_{0,L} = \text{Gamma} \left( m_L, \nu_i \frac{\lambda_{i,L}^c}{m_L} \right) \\
C_{0,N} &= \text{Gamma} \left( m_N M, \gamma_T^{-1} \nu_0 \frac{\lambda_{0,N}^c}{m_N} \right) ; \quad B_{0,N} = \text{Gamma} \left( m_N, \nu_i \frac{\lambda_{i,N}^c}{m_N} \right)
\end{align*}
\]

with \( \lambda_{i,L}^c = \beta_{i,L} \chi_{i,L} \Omega_L \) and \( \lambda_{i,N}^c = \beta_{i,N} \chi_{i} \Omega_N \).

Considering these modifications, (27) will have the following expression

\[
\int_{\sum_{i=1}^K z_i+1}^{\infty} f_R(r) dr = P_{\text{LOS}}(d_0) \frac{\Gamma \left( m_L M, \left( \sum_{i=1}^K z_i + 1 \right) T_L \right)}{\Gamma \left( m_L M \right)} + (1 - P_{\text{LOS}}(d_0))
\]

\[
\times \frac{\Gamma \left( m_N M, \left( \sum_{i=1}^K z_i + 1 \right) T_N \right)}{\Gamma \left( m_N M \right)}
\]

where \( T_L = \gamma_T m_L / (\nu_0 \lambda_{0,L}^c) \) and \( T_N = \gamma_T m_N / (\nu_0 \lambda_{0,N}^c) \).

Considering this, we use the same steps as in theorem 2 to derive the outage probability in the presence of blockages as in (25) and (27), we added the term which corresponds to case when the SM is in NLOS with the gateway. This addition has no effect in the derivation of the outage probability.

**APPENDIX F**

**PROOF OF COROLLARY 3**

Starting by the expression of the outage probability in corollary 2

\[
P_{\text{out}}(\gamma_T, \nu_0, \nu) = P_{\text{LOS}}(d_0) \ P_{\text{out,L}}(\gamma_T, \nu_0, \nu) + (1 - P_{\text{LOS}}(d_0)) \ P_{\text{out,N}}(\gamma_T, \nu_0, \nu)
\]

In the high SNR regime, \( \nu_0 \to \infty, \frac{\gamma_T m_L}{\nu_0 \lambda_{0,L}} \to 0 \) and \( \frac{\gamma_T m_N}{\nu_0 \lambda_{0,N}} \to 0 \). In this case, \( P_{\text{out}}^\infty(\gamma_T, \nu_0, \nu) \) and \( P_{\text{out}}^\infty(\gamma_T, \nu_0, \nu) \) correspond to the first terms of each first sum (the first term of the sum corresponds to the index \( k = 0 \)) from \( P_{\text{out,L}}(\gamma_T, \nu_0, \nu) \) and \( P_{\text{out,N}}(\gamma_T, \nu_0, \nu) \). The outage probability,
in the high SNR regime, is expressed as follows

\[ P_{\text{out}}^\infty (\gamma_T, \nu_0, \nu) = P_{\text{LOS}} (d_0) P_{\text{out}, L}^\infty (\gamma_T, \nu_0, \nu) + (1 - P_{\text{LOS}} (d_0)) P_{\text{out}, N}^\infty (\gamma_T, \nu_0, \nu) \]  \hspace{1cm} (66)

where \( P_{\text{out}, L}^\infty (\gamma_T, \nu_0, \nu) \) and \( P_{\text{out}, N}^\infty (\gamma_T, \nu_0, \nu) \) are given in corollary 3. Next, we are going to prove that \( P_{\text{LOS}} (d_0) P_{\text{out}, L}^\infty (\gamma_T, \nu_0, \nu) \ll (1 - P_{\text{LOS}} (d_0)) P_{\text{out}, N}^\infty (\gamma_T, \nu_0, \nu) \) as follows

\[
\lim_{\nu_0 \to \infty} \frac{P_{\text{LOS}} (d_0) A_L (\nu)}{(1 - P_{\text{LOS}} (d_0)) A_N (\nu)} = \lim_{\nu_0 \to \infty} \frac{P_{\text{LOS}} (d_0) A_L (\nu)}{(1 - P_{\text{LOS}} (d_0)) A_N (\nu)} \frac{\left( \gamma_T m_L \right)^{Mm_L} \left( \frac{\nu_0 \lambda_{0,N}}{\gamma_T m_N} \right)^{Mm_N}}{(\gamma_T m_L)^{Mm_L} \left( \frac{\lambda_{0,N}}{\gamma_T m_N} \right)^{Mm_N} \nu_0^{M(m_N - m_L)}}
\]

where

\[
A_N (\nu) = \frac{f_{\sigma_+, \nu} (Mm_N m_L + 1)}{\Gamma (m_N M + 1)} \sum_{t=0}^{Mm_N} \binom{Mm_N}{t} \sum_{i=1}^K \prod_{i=1}^K (1 - p_i) \delta_{t_i} \frac{p_i P_{\text{LOS}} (d_i) \Gamma (t_i + m_L)}{t_i! \Gamma (m_L)} 
\]

\[
\times \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i} f_{\sigma_-, \nu} (t_i) + \frac{p_i (1 - P_{\text{LOS}} (d_i)) \Gamma (t_i + m_N)}{t_i! \Gamma (m_N)} \left( \frac{\nu_i \lambda_{i,N}}{m_N} \right)^{t_i} f_{\sigma_-, \nu} (t_i) \]
\]

(69)

\[
A_L (\nu) = \frac{f_{\sigma_+, \nu} (Mm_L m_L + 1)}{\Gamma (m_L M + 1)} \sum_{t=0}^{Mm_L} \binom{Mm_L}{t} \sum_{i=1}^K \prod_{i=1}^K (1 - p_i) \delta_{t_i} \frac{p_i P_{\text{LOS}} (d_i) \Gamma (t_i + m_L)}{t_i! \Gamma (m_L)} 
\]

\[
\times \left( \frac{\nu_i \lambda_{i,L}}{m_L} \right)^{t_i} f_{\sigma_-, \nu} (t_i) + \frac{p_i (1 - P_{\text{LOS}} (d_i)) \Gamma (t_i + m_N)}{t_i! \Gamma (m_N)} \left( \frac{\nu_i \lambda_{i,N}}{m_N} \right)^{t_i} f_{\sigma_-, \nu} (t_i) \]
\]

(70)

Using the fact that \( m_N < m_L \) \((m_N - m_L < 0)\), we have \( \lim_{\nu_0 \to \infty} \nu_0^{M(m_N - m_L)} = 0 \). Hence,

\[
\lim_{\nu_0 \to \infty} \frac{P_{\text{LOS}} (d_0) P_{\text{out}, L}^\infty (\gamma_T, \nu_0, \nu)}{(1 - P_{\text{LOS}} (d_0)) P_{\text{out}, N}^\infty (\gamma_T, \nu_0, \nu)} = 0
\]

(71)

Therefore,

\[
P_{\text{out}}^\infty (\gamma_T, \nu_0, \nu) \leq (1 - P_{\text{LOS}} (d_0)) P_{\text{out}, N}^\infty (\gamma_T, \nu_0, \nu),
\]

(72)
REFERENCES


